Chapter 5 Cost-Benefit Analysis

5.1 Introduction

The principle of cost-benefit analysis is straightforward. Any investment project can be viewed as representing a perturbation of the economy from what it would have been had the project not been undertaken. To evaluate whether the project should be undertaken, we need to look at the levels of consumption of all individuals of all commodities at all dates, under the two different situations.

If all individuals are better off with the project than without it, then it should be adopted (if there is an individualistic social welfare function); if all individuals are worse off, then it should be rejected. If some individuals are better off, and some worse off, whether we should adopt it depends on how we weight the gains and losses of different individuals.

Although this is obviously the "correct" procedure to follow in evaluating projects, it is not a practical one; the problem of cost-benefit analysis is simply whether we can find reasonable short cuts. In particular, we are presumed to have good information concerning the *direct costs and benefits* of a project; the question is whether there is any simple way of relating *the total effects* (the total changes in the vectors of consumption) to *the direct effects*. Thus, in the case of the choice of discount rate, there is a trivial sense in which we would always wish to use *the social rate of time preference* for evaluating benefits and costs accruing in different periods. This however applies to total effects, and there is no reason to believe that there are simply proportional to the direct effects that are observed.

If the ratio of total effects to direct effects changes systematically over time, then we would not wish to use the social rate of time discount in evaluating a project when looking only at direct costs and benefits.

In a first-best world, with no distortions and full scope for lump-sum redistributive taxation, if a project is "profitable" on the basis of its direct effects *using market prices*, then – with an individualistic social welfare function – it is socially desirable.

The problem of finding the correct *shadow prices* for cost-benefit analysis arises from *the existence of market imperfections and failures*; it is concerned with situations where one cannot necessarily infer social desirability on the basis of the profitability of the project. In the case of *the social rate of discount*, the difficulties stem from differences between *the private rate of return* and *the rate at which society can transfer resources between periods*.

The former is equal, in a competitive model, to *the marginal physical rate of transformation* of output in one period into output in the next. The latter is the rate at which the government can make the transfer, or what we refer to as *the marginal economic rate of transformation*. In applying this approach, we need to begin with the reasons why a first-best cannot be attained. This depends on the initial sources of *market failure*, and on the extent to which government policy instruments can be employed to approach the first-best (i.e. if the market solution is socially optimal, then there is no need for government intervention).

Cost-benefit analysis might be carried out to assist an international or bilateral (i.e. foreign) agency or a national public sector agency.

A foreign agency does not usually undertake projects, it finances them. The decision to finance a project, its rationale, and the processes by which it is reached are the responsibility of the foreign agency, not that of the borrower. If the financing decision is based on cost-benefit analysis, the value judgements used are also the responsibility of that agency.

5.2 Fundamental Issues Related to Willingness-to-Pay[1]

Three sets of fundamental issues arise with respect to the interpretation of *willingness-to-pay* as a measure of benefits in the assessment of the efficiency of policies. First, a theoretical limitation in the aggregation of willingness-to-pay amounts across individuals opens the possibility that the net benefits criterion will not lead to fully satisfactory rankings of policies. Second, normative issues arise because of the dependence of willingness-to-pay on the distribution of wealth in society. Third, normative issues also arise with respect to the issue of *standing*, which concerns whose willingness-to-pay counts in the aggregation of benefits.

The Theoretical Limitation of Willingness-to-Pay as a Basis for Social Orderings

Although using net benefits as a basis for choosing efficient Public policies is intuitively appealing, its implementation through the aggregation of the willingness-to-pay amount of the members of society confronts a fundamental theoretical limitation: Ranking policies in terms of net benefits does not guarantee a *transitive* social ordering of the policies.

A transitive ordering requires that if X is preferred to Y, and Y is preferred to Z, then X is preferred to Z. The logic of transitivity seems so clear that it is usually taken as an axiom of rationality in the preferences of individuals. We would certainly be skeptical about the mental state of someone who tells us he prefers apples to oranges, and he prefers oranges to peaches, but he prefers peaches to apples. This violation of transitivity implies a cyclical and, therefore, ambiguous ordering of the alternatives: the ordering – apples, oranges, peaches, apples – leaves us uncertain as to whether this person ranks apples lowest or highest in his preferences.

[1] This section is drawn from Boardman, Greenberg, Vining and Weimer (2000), pp.32-8.

Clearly, transitivity is a desirable property of any preference ordering.

If every member of a society has transitive preferences, does it follow that reasonable procedures for aggregating their preferences will always produce a transitive social ordering? An example makes clear that the answer is no. Consider a very common aggregation procedure: majority rule voting over pairs of alternatives. Imagine that society consists of three voters who have preferences over three alternatives, *X*, *Y*, and *Z* as displayed in Table 5.1. Specifically, voter 1 prefers *X* to *Y* to *Z*; voter 2 prefers *Z* to *X* to *Y*; and voter 3 prefers *Y* to *Z* to *X*. If the voters express their sincere preferences in each round of voting, then we would find that given the choice between *X* and *Y*, a majority of voters (voters 1 and 2) would vote for *X* because they each prefer it to *Y*. Similarly, given the choice between *Y* and *Z*, a majority would vote for *Y*. Yet in a choice between *X* and *Z*, a majority would vote for *Z*. Thus, the implied social ordering is intransitive because *X* is preferred to *Y*, *Y* is preferred to *Z*, but *Z* is preferred to *X*!

Table 5.1 Cyclical Social Preferences under Pairwise Majority Rule Voting

Preference Ordering	Voter 1	Voter 2	Voter 3
First Choice	*X*	*Z*	*Y*
Second Choice	*Y*	*X*	*Z*
Third Choice	*Z*	*Y*	*X*

Pairwise Voting Outcomes: *X* versus *Y*, *X* wins; *Y* versus *X*, *Y* wins: *X* versus *Z*, *Z* wins.
Implied Social Ordering: *X* is preferred to *Y*, *Y* is preferred to *Z*, but *Z* is preferred to *X*!

Is the possibility of obtaining an intransitive social ordering peculiar to the use of pairwise majority rule voting to produce rankings alternative? Surprisingly, it can result from any rule for creating a social ordering that satisfies certain minimal requirements. We cannot expect any rule for creating a social ranking of policy alternatives to be fully satisfactory. In 1951, Kenneth Arrow proved that any *social choice rule* that satisfies a basic set of fairness conditions could produce an intransitive social ordering. Arrow's *General Possibility Theorem* applies to any rule for choice in which two or more persons must select a policy from among three or more alternatives. It requires any such scheme to satisfy at least the following conditions to be considered fair: First, each person is allowed to have any transitive preferences over the possible policy alternatives (*axiom of unrestricted domain*). Second, if one alternative is unanimously preferred to a second, then the rule for choice will not select the second (*axiom of Pareto choice*). Third, the ranking of any two alternatives should not depend on what other alternatives are available (*axiom of independence*). Fourth, the rule must not allow any one person dictatorial power to impose his or her preferences as the social ordering (*axiom of nondictatorship*). Arrow's theorem states that any fair rule for choice (one htat satisfies the four preceding axioms) will not guarantee a transitive *social ordering* of policy alternatives. That is, it is possible that individual preferences are such that the social ordering will be cyclical. Thus, unless the net benefit rule, which is a social choice rule, violates one of the axioms, it cannot guarantee a transitive social ordering of policies.

In order to ensure that the use of willingness-to-pay in the implementation of the net benefit rule will produce a transitive social ordering of policies, some restrictions, violating the axiom of unrestricted domain, must be placed on the preferences that individuals are allowed to hold. Economic models commonly assume that individual preferences are represented by utility functions (numerical representations of preference orderings) that exhibit positive but declining marginal utility – other things equal, incremental consumption of any good increases utility but not by as much as the previous incremental unit. Unfortunately, this relatively weak restriction of the domain of preferences (it rules out preference that cannot be represented by such utility functions) is not enough to guarantee that the net benefit rule based on willingness-to-pay will *always* produce a transitive social ordering. Two additional restrictions are required: The utility functions of individuals must be such that the individual demand curves that they imply can be aggregated into a market demand curve with the sum of individual incomes as an argument; and all individuals must face the same set of prices. The first restriction is quite strong in that it requires every individual's demand for each good to increase linearly with increasing income and have the same rate of increase for each individual. The second restriction, generally satisfied when all goods are traded in markets, may be violated when policies allocate quantities of goods to individuals who cannot resell them in markets.

The necessity of restricting the allowed preferences of individuals to guarantee a transitive social ordering from the use of willingness-to-pay in the implementation of the net benefits criterion makes clear that it is an imperfect criterion for assessing the relative efficiency of alternative policies. Of course, analysis can avoid this theoretical problem by assuming that the preferences of individual consumers conform to restrictive assumptions consistent with the existence of an appropriate aggregate demand function. Alternatively, analysts can avoid the problem by assuming that policies affect the price of only a single good so that choice is over a single dimension. Indeed, analysts seeking to estimate willingness-to-pay typically work with an aggregate, or market, demand schedule for a single good, implicitly assuming away price effects in the markets for other goods.

Despite its theoretical imperfection as a measure of efficiency, willingness-to-pay is an intuitively appealing and practical concept for guiding the implementation of the net benefits criterion. As discussed next, however, its dependence on the distribution of wealth raises a serious normative concern about its use.

Dependence of Willingness-to-Pay on the Distribution of Wealth

The willingness of a person to pay to obtain a desired policy impact will tend to be higher the greater the wealth that she or he has available. Consequently, the sum of the willingness of persons to pay, the benefit measure in CBA, depends on their levels of wealth. If the distribution of wealth in society were to be changed, then it would be likely that the sum of individuals' willingness-to-pay amounts would change as well, perhaps altering the ranking of alternative policies in terms of their net benefits.

The dependence of net benefits on the distribution of wealth would not pose a conceptual problem if losers from adopted policies were *actually* compensated so that the adopted polices would produce actual, rather than potential, Pareto improvements. From a utilitarian perspective, Pareto improvement guarantees that the sum of utilities of individuals in society increases. In application of the potential Pareto principle, however, it is possible that an adopted policy could actually lower the sum of utilities if people with different levels of wealth had different *marginal utilities of money*. As an illustration, consider a policy that gives $10 of benefits to a person with high wealth and inflicts $9 of costs on a person with low wealth. If the low-wealth person's marginal utility of money is higher than that of the high-wealth person, then it is possible that the utility loss of the low-wealth person could outweigh the utility gain of the high-wealth person. Thus, although the Pareto principle allows us to avoid interpersonal utility comparisons by guaranteeing increases in aggregate utility for policies with positive net benefits, the potential Pareto principle does not do so.

The implication of the dependence of willingness-to-pay on wealth is that the justification for the potential Pareto principle weakens for policies that concentrate costs and benefits on different wealth groups. Policies with positive net benefits that concentrate costs on low-wealth groups may not increase aggregate utility; moreover, policies with negative net benefits that concentrate benefits on low-wealth groups may not decrease aggregate utility. However, if the potential Pareto principle is consistently applied, and adopted policies do not produce consistent losers or winners, then the over-all effects of the policies taken together will tend to make everyone better off. Hence, concerns about reductions in aggregate utility would be unfounded.

Critics of CBA sometimes question the validity of the concept of Pareto efficiency itself because it depends on the status quo distribution of wealth. Returning to Figure 5.1, note that the location of the Pareto frontier would change if the location of the status quo point were changed. Some have advocated the formulation of a social welfare function that maps the utility, wealth, or consumption of all individuals in society into an index that ranks alternative distributions of goods. In this broader framework incorporating distributional values, an efficient policy is one that maximizes the value of the social welfare function. But how does society determine the social welfare function? Unfortunately, Arrow's General Possibility Theorem, as well as practical difficulties in obtaining needed information, preclude the formulation of a social welfare function through any fair collective choice procedure. In practice, it must, therefore, be provided subjectively by the analyst. We believe that it is usually better to keep the subjective distributional values of analysts explicit by comparing policies both in terms of efficiency and the selected distributional criteria. As an alternative, analysts can report net benefits by wealth or income group as well as for society as a whole.

Dependence of Net Benefits on Assumptions about Standing

The question of whose willingness-to-pay should count in th aggregation of net benefits has come to be known as the issue of standing. It has immediate practical importance in at least three contexts: the jurisdictional definition of society, the exclusion of socially unacceptable preferences, and the inclusion of the preferences of future generations. A recognition of social constraints, rights, and duties often helps answer the question of standing.

Juristictional Definition of Society The most inclusive definition of society encompasses all people, no matter where they live or to which government they owe allegiance. Analysts working for the United Nations or some other international organization might very well adopt such a universalistic, or global, perspective. Yet for purposes of CBA, most analysts define society at the national level. The basis for this restriction in jurisdiction is the notion that the citizens of a country share a common constitution, formal or informal, that sets out fundamental values and rules for making collective choices. In a sense, they consent to being a society. Furthermore, they accept that the citizens of other countries have their own constitutions that make them distinct societies.

The distinction between universal and national jurisdiction becomes relevant in the evaluation of policies whose impacts spill over national borders. For example, if U.S. analysts adopt the national-level jurisdiction as defining society, then they would not attempt to measure the willingness of Canadian residents to pay to avoid pollution originating in the United States that exacerbates acid rain in Canada. Of course, the willingness of U.S. citizens to pay to reduce acid rain in Canada should be included in the CBA, though in practice it would be very difficult to measure.

A similar issue arises with respect to subnational units of government. As an illustration, consider a city that is deciding whether to build a convention center. Assume that a CBA from the national perspective (giving standing to everyone in the country) predicts that the project will generate $1 million in benefits (which all accrue to city residents), $2 million in costs (which are also borne by city residents) and, therefore, negative $1 million in net benefits (or $1 million in net costs). Also assume, however, that through an intergovernmental grants program, the national government will repay the city $2 million for costs resulting from this particular project. The grant appears to the city residents as a $2 million benefit off-setting $2 million in local costs. Thus, from the perspective of the city, the convention center generates $1 million in net benefits rather than $1 million in net costs.

One can make an argument that the city should treat its residents as the relevant society and, hence, should not give standing to nonresidents. The city government has a charter to promote the welfare of its residents. The city by itself can do relatively little to affect national policy – even if it does not take advantage of all the opportunities offered by the national government, other cities probably will. Furthermore, analysts who do not adopt the city's perspective, but instead employ only the broader national perspective, risk losing influence, a possibility of special concern to analysts who earn their living by giving advice to the city.

Adopting the subnational perspective, however, makes CBA a less valuable decision rule for public policy. We believe that analysts should generally conduct CBA from at least the national perspective. They may, of course, also conduct a parallel CBA from the subnational perspective as a response the narrower interest of their clients. If major impacts spill over national borders, then the CBA should be done from the global as well as the national perspective.

Jurisdictional Membership Deciding the jurisdictional definition of society leaves open a number of questions about who should be counted as members of the jurisdiction. For example, almost all analysts agree that citizens of their country living abroad should have standing. With respect to noncitizens in their country, most analysts would probably give standing to hose who are in the country legally. Less consensus exists with respect to the standing of other categories of people: Should illegal aliens have standing? What about the children of illegal aliens?

One sorce of guidance for answering these sorts of questions is the system of legally defined rights. For example, a ruling by the courts that the children of illegal aliens are entitled to access publicly funded education might encourage the analyst to give these children standing in CBA. Reliance on legally defined rights to determine standing, however, is not always morally acceptable. It would not have been right to deny standing in CBA to slaves in the antebellum United States, nonwhites in apartheid South Africa, or Jews in Nazi Germany simply because they lacked legal rights. Therefore, legal rights alone cannot fully resolve the issue of standing in CBA – they provide a presumption, but one that analysts may sometimes have an ethical responsibility to challenge. Democratic polities usually provide mechanisms for challenging such presumptions, but often with personal cost to individual analysts.

One other issue of membership deserves brief mention. CBA is anthropocentric. Only the willingness-to-pay of people counts. Neither flora nor fauna have standing. That is not to say that their "interests" have no representation. Many people are willing to pay to preserve species, and some are even willing to pay to preserve individual animals or plants. It is conceptually correct within the CBA framework to take account of these willingness-to-pay amounts, though effectively doing so is very often beyond our analytical reach.

Exclusion of Socially Unacceptable Preferences People sometimes hold preferences that society seeks to suppress through widely supported legal sanctions. For instance, though some people would be willing to pay for the opportunity to have sexual relations with children, most countries attempt to thwart the expression of such preferences through very strict criminal penalties. Should such socially unacceptable preferences be given standing in CBA?

One approach to answering this question adds duties and prohibitions to legal rights as sources of guidance about social values. Together they can be thought of as social constraints that should be taken into account in CBA just as the analyst takes account of physical and budgetary constraints. Clear and widely accepted legal sanctions may help identify preferences that should not have standing.

An important application arises in estimating the net benefits of policies that are intended to reduce the amount of criminal behavior in society. Some analysts count reductions in the monetary returns to crime as a cost borne by criminals, offsetting the benefits of reduced criminal activity enjoyed by their victims. As the returns from crime are illegal and widely viewed as wrong, however, the social constraint perspective argues against treating them in this manner.

The issue of the standing of preferences can be especially difficult for analysts to resolve when they are dealing with foreign cultures. Consider, for instance, the CBA of a program to bring water to poor communities in Haiti. Analysts found that husbands had a negative willingness-to-pay for the time that their wives saved from easier access to water. By contemporary standards in most urban settings, people would generally regard these preferences as unworthy. Yet in the cultural context of rural Haiti at the time, they were quite consistent with prevailing norms. Should these preferences of husbands have standing? In practice, lack of data to estimate willingness-to-pay amounts for this sort of impact usually spares analysts from having to answer such difficult questions.

Inclusion of the Preferences of Future Generations Some policies adopted today, such as the disposal of nuclear wastes or the restoration of wilderness areas, may have impacts on people not yet born. Though we believe that these people should have standing in CBA, there is no way to measure their willingness-to-pay directly because they are not yet here to express it. How serious a problem does this pose for CBA?

The absence of direct measures of the willingness of future generations to pay for policy impacts is unlikely to be serious in most circumstances for two reasons. First, Because few policies involve impacts that appear only in the far future, the willingness-to-pay of people alive today can be used to predict how future generations will value them. Second, as most people alive today care about the well-being of their children, grandchildren, and great-grandchildren, whether or not they have yet been born, they are likely to include the interests of these generations to some extent in their own valuations of impacts. Indeed, because people cannot predict with certainty the place that their future offspring will hold in society, they are likely to take a very broad view of future impacts.

5.3 Project Appraisal in Developing Countries

Applications of cost-benefit analysis are not restricted to developing countries. As an early example, it is applied to the formation of public policy in the 1936 United States Flood control Act.

Industrialized countries all now have procedures for evaluating the costs and benefits of *road and rail construction*, for quantifying the effects of *noise and air pollution*, and for estimating the net benefits of public sector social outputs such as *health and education*.

In the literature on developing countries, however, the range of subjects discussed under the heading of project appraisal is far more extensive.

There are several reasons why cost-benefit analysis and the theory of project appraisal have been more fully developed in the context of developing economies.

1) A need for cost-benefit analysis arises in cases of market failure

When markets are *distorted, or do not exist*, the existing structure of prices cannot be relied upon to allocate resources in an efficient manner, in such cases, these may be grounds for public sector intervention.

All these departures from the perfectly competitive model provide a rationale for cost-benefit analysis. Distorted or missing markets constitute arguments for public sector intervention on grounds of efficiency.

2) A concern for distribution has also been a powerful force in stimulating interest in policies for economic development.

   The theory of project appraisal in developing countries has been concerned with both these aspects of the *inequitable distribution of income*. Suggestions for incorporating weights to favor projects which reduce income inequality can be seen in many of the project appraisal manuals and the whole rationale for project appraisal methodologies is that of *promoting economic growth in poor countries*.

3) As we saw in development finance, *investment* is seen as central to the process of raising and maintaining living standards. The problem is to find funds for investment: *raising savings* is seen as the key to growth.

Given the low levels of *per capita* incomes, it was not always easy to raise domestic savings and it was also widely believed that exports could not be increased sufficiently to generate the funds needed to purchase foreign machinery and technology. In the face of this 'two-gap' model of poor countries, governments are encouraged to look abroad for *foreign aid*.

Of course, how to use foreign aid for economic development is the key issue. Before deciding any public investment, we need project appraisal.

Two most influential books on the subject of project appraisal in developing countries were published: *The UNIDO Guidelines for Project Evaluation*, Dasgupta, Sen and Marglin (1972) and *Project Appraisal and Planning for Developing Countries*, Little and Mirrlees (1974). These two works provide a novel and controversial methodology for the appraisal of public sector projects in less developed countries.

These works provide fundamental rules for project appraisals (set up the standard for the modern approach);

First, both works emphasize *the micro-foundations of investment appraisal*: the need to use *prices* to ensure that the efficient use of resources is at the forefront of the analysis.

Second, *country should not ignore any actual or potential gains from trade: use the world prices for investment appraisal in order to ensure that these gains were not overlooked*.

The concept of a 'project' has varied over time with the shifting views of the role of investment. The original project appraisal methodology was typically concerned with *physical investment in new plant*. The concept was widened to cover 'programs' – rural development schemes with substantial *training and educational* inputs have become important in recent decades.

More recently, *the structural adjustment programs of the World Bank and IMF* have been largely concerned with policies intended to reduce perceived market distortions by altering relative prices.

5.3 Welfare Objectives

It is essential to emphasize that cost-benefit analysis and all policy recommendations following from the cost-benefit approach can only be interpreted in terms of *the chosen social objective function* (in most case, it is to *increase levels of per capita consumption*).

It can be extended to include *changes in the distribution of consumption over time* and include implicitly or explicitly *a judgement about the distribution of consumption gains between individuals at any given point in time*.

Consumption is not restricted to those goods, which can be purchased by individuals; *public goods* provided by the state and *environmental benefits*, such as clean air or uncontaminated water are also included.

A lot of alternative for social welfare function, and social objectives can be identified.

1) national self-respect
2) shift in the class structure of society
3) *UN Human Development Report 1992* introduced a human development index (HDI) combines indicators of *per capita national income*, *life-expectancy* and *educational attainment.*
4) Dasgupta's 'social well-being function' includes *per capita income, life-expectancy, infant mortality, adult literacy, an index of political rights and an index of civil rights*.

5.4 Private and Social Approaches to Investment Appraisal[*]

In fact, the procedures adopted by the cost-benefit analyst are very similar to what the private sector does. Taking as the object of the exercise an increase in *social profitability*, the costs and benefits from a proposed project in the public sector – a state enterprise, a road, an irrigation project – will be assessed, using many of the same techniques as those used by the private sector. The essential difference between the procedures follows from the use of *social profitability* (defined in terms of social welfare) which will lead the planner of projects in the public sector to use at least *some prices which have been especially constructed to reflect the chosen social objectives*. Why not use the market price?

There are a number of reasons why prevailing market prices may fail to measure either *the marginal social cost* or *the marginal social benefit* or *both*.

First, any kind of market distortion drives a wedge between the price to the producer and the

[*] This section is drawn heavily from Dinwiddy and Teal (1996, Chapter 5).

price to the consumer; imperfectly competitive markets or indirect taxes and subsidies mean that there is not just one price available for project appraisal. There may be economic outputs without market price (e.g. *soil erosion, preserving a rainforest*).

For all these reasons, the cost-benefit analysis may be required to supply alternative prices to be used when current market prices either do not exist, or for some other reasons are thought not to represent the social opportunity cost of resources and/or the value of the social benefits to be generated by the project. Such prices are usually referred to as "*shadow prices*" or "*accounting prices*".

Control Variables and Shadow Prices

A shadow price measures the marginal effect on social welfare of a change in a government control variable. A control variable is any decision variable under public sector control.

In the context of a project, the purchased inputs, whether labor, capital, raw materials or intermediate inputs, are *government control variables*; so are the outputs of the projects.

In the context of pricing policies, any *tax* or *subsidy* or *administered price* can be regarded as a government control variable.

In order to show how *shadow prices* can be derived for control variables in terms of the social welfare function, we use the single representative household model to measure welfare by

$$U = U(C_1, C_2, C_3 \cdots) \tag{5.1}$$

where $C_i \, (i = 1,2 \cdots n)$ are the level of consumption of different commodities. The differential of this measure is

$$dU = U_1 dC_1 + U_2 dC_2 + U_3 dC_3 \cdots + U_n dC_n \tag{5.2}$$

where $U_i = \partial U / \partial C_i$ the marginal utility of commodity *i*. Dividing by λ, the marginal utility of income, gives the following measure of welfare change *dW*:

$$\partial W = \frac{dU}{\lambda} = P_1 dC_1 + P_2 dC_2 + \cdots + P_n dC_n \tag{5.3}$$

In the theory of shadow pricing, it is usual to assume that the projects being evaluated have an impact, which is only marginal from the point of view of the economy as a whole. With this assumption, changes in the variable λ which measures the marginal utility of income can be ignored, and the right-hand side of (5.3) can be used as *a differential measure of welfare change* in which changes in consumption levels are evaluated around an initial equilibrium in terms of *their base level prices*.

It must be emphasized that *the prices P_i are the actual market prices facing the consumer*. They may be distorted prices, including taxes, for example; their welfare significance comes from the fact that the relative prices reflect the relative marginal utilities of the consumer.

The social welfare function can be adopted to provide *a measure of welfare change over time*. If we reinterpret (5.2), so that the dC_i refer, not to different commodities, but to the consumption of a composite bundle of commodities at different points in time, *the relevant intertemporal prices* will be expressed *in terms of the rate at which the consumer discounts the value of consumption in the future in relation to his/her consumption in the present*.

$$\partial W = dC_0 + \frac{dC_1}{(1+i)} + \frac{dC_2}{(1+i)^2} + \cdots + \frac{dC_T}{(1+i)^T} \tag{5.4}$$

where $t = 0,1,\cdots T$ time period, i = the consumer's rate of discount, the discount factor $1/(1+i)^t$ is the price of future consumption at period *t* in terms of present consumption.

Although (5.3) is the fundamental measure of welfare change and can be used to evaluate the welfare effects of projects, *it is not in the form needed to identify the shadow prices to be used by the public sector in their cost-benefit analysis*. For this, we need *welfare measure in terms of the government control variables,* denoted by G_i.

A shadow price measures the marginal social cost and/or benefit of a change in government control variable. If, for example, let *G* represent a public sector project input or output, the shadow price or utility value, of this input/output will be given by $\partial W/\partial G_i$, *the marginal change in welfare brought about by the increased use of the input, or the additional output*. To identify these shadow prices we have to convert the welfare measure (5.3) into an equivalent measure having the general form

$$\partial W = \frac{\partial W}{\partial G_1} dG_1 + \frac{\partial W}{\partial G_2} dG_2 + \cdots + \frac{\partial W}{\partial G_k} dG_k \tag{5.5}$$

where the partial derivative $\partial W/\partial G_i \; (i = 1,2,\cdots k)$ *provide the required shadow prices for the k public sector inputs/outputs*.

Optimum and Sub-optimum in Cost-Benefit Analysis

There is a prevalent and unfortunate misunderstanding about the nature of cost-benefit analysis, which sometimes leads students to think that the theoretical object of cost-benefit analysis is to describe socially optimal states of affairs. That is not the view of the subject portrayed here. We present *cost-benefit analysis as a tool for deciding whether or not a particular project or policy could be expected to lead to an improvement in social welfare*.

The theory of the second best states that *if only one market distortion exists its removal will necessarily constitute a welfare improvement, but that in an economy with more than one distortion correcting for any one particular market failure will not necessarily be welfare improving*.

Clearly if welfare economics is perceived to be a set of prescriptions for achieving social optima the problem presented by the theory of the second best is serious; when, however, cost-benefit analysis is seen as a set of recommendations for the measurement of welfare change at a given, sub-optimal state of the economy, this problem does not arise.

In the cost-benefit approach, certain distortions, which may well result from a sub-optimal policy design, are taken for granted when projects or policies are assessed: Recommendations from a cost-benefit analysis are intended to enable investors in the public sector to know whether a particular project, undertaken in a given economic environment which includes any currently irremovable distortions, will lead to a welfare improvement. Policy prescriptions based on cost-benefit procedures are not invalidated by the theory of the second best.

5.5 Investment Criteria

The costs and benefits from an investment project will extend from the present into the future.

The largest costs are likely to be incurred at the beginning of the period when *the new plant is constructed* and *machinery installed*; but there will also be future costs, reflecting payments for the project inputs over its projected lifetime – *the cost of materials* used in the production process, *the cost of factor inputs* incurred in operating and maintaining the plant, and *possible replacement costs* for some items.

The stream of revenues from the project will normally begin at some future date, after the initial construction and start-up period. The difference between income and expenditures will constitute the net profit or net benefit of the project.

Much of the difficulty in any appraisal will be in trying to *estimate these future costs and revenues*, we abstract from that problem and assume that figures have been provided by the engineers and economists working on the project. The next problem to be addressed is that of making costs and benefits which occur at different points in time commensurate. This is done by *discounting*.

The principle of discounting is based on the fact that a sum of money deposited in a bank or other financial institution now at a given rate of interest will increase in value: If P is the present value of the sum, i the rate or interest, t the number of time periods and A the future value. The future value can be calculated by means of the compound growth formula

$$A = P(1+i)^t \tag{5.6}$$

The present value of a sum of money worth A in the future can be found by discounting;

$$P = \frac{A}{(1+i)^t} \tag{5.7}$$

i is known as *the discount rate* [2]and

$$\frac{1}{(1+i)^t}$$

as *the discount factor*. The discount factor gives the price of A in terms of its present value. Given a stream of values $A_0, A_1, A_2, \cdots$ occurring at $t = 0,1,2,\cdots$ the present value (PV) can be calculated by

$$PV = A_0 + \frac{A_1}{(1+i)} + \frac{A_2}{(1+i)^2} + \cdots \tag{5.8}$$

In the context of cost-benefit analysis, the problem is to discount *the stream of net benefits,* where the net benefits are defined as *the difference between the benefits and the costs for each year of the project*. The resulting value is the net present value of the project, *NPV* such that,

$$NPV = NB_0 + \frac{NB_1}{(1+i)} + \frac{NB_2}{(1+i)^2} + \frac{NB_3}{(1+i)^3} + \cdots + \frac{NB_T}{(1+i)^T} \tag{5.9}$$

[2] The discount rates used in US government agencies in 1969 were as follows: (1) defense 10-12%, (2) agency for international development 8-12%, (3) development of the interior 6-12% (energy programs), 3-6% (all other projects), (4) health, education and welfare 0-10%, (5) Tennessee Valley Authority, Department of Agriculture, Office of Economic Opportunity, Department of Transportation all<5%, and (6) all other agencies, no discounting.

$$NB_t = Benefit_t - Cost_t$$

There are two questions to be asked. First, *does the project represent a good use of the funds employed*? Second, *is the project under consideration to be preferred to other projects*, which could be carried out with the funds available?

Both these questions can, in principle, be answered by using *the NPV criterion*. An investment is worthwhile if the *NPV* is positive: that is, if *NPV* >0.

One project (A) is to be preferred to another project (B) if *NPV* (*A*) > *NPV* (*B*) with the same fund (cost) available.

It should be noted that *both of these criteria are based on the principle of opportunity cost*. When calculating the *NPV* of a project at a given rate of interest *i* and using the criterion *NPV* >0, we are comparing the returns from the investment project *with the returns from depositing the funds in a bank, or lending it at a rate i to another institution*.

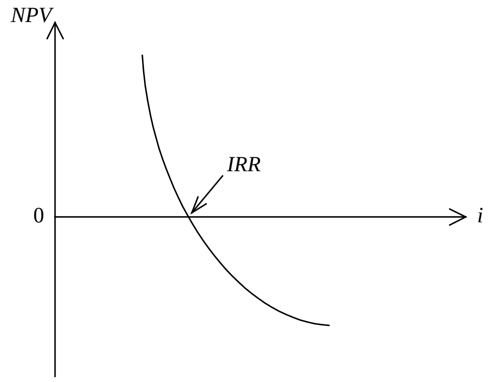
If the *NPV* of the project is zero or negative, then the funds could be equally well, or better used elsewhere.

Similarly the *NPV(B) will represent the opportunity cost of project A, NPV*(*A*) – the next best use of funds if A is not undertaken.

Internal Rate of Return (IRR)

The internal rate of return is defined such that the discount rate satisfies that *NPV* is equal to zero. The relationship between *NPV* and *IRR* of a project can be represented graphically.

Figure 5.1



The use of the *IRR* as an investment criterion is based on a comparison between the *IRR* and the discount rate which measures the opportunity cost of the funds bound up in the project. The rule is to proceed if *IRR*>*i* (The modern version of this idea is *Tobins' q theory* of investment).

For many project it does not matter whether the criterion based on the *NPV* or that based on the *IRR* is used. Occasionally, the *IRR* will not have a unique value; This will not happen in those projects where initial expenditures are succeeded by a stream of net benefits; but if substantial net costs are anticipated at a future date, multiple values of the *IRR* can result and the rule based on *IRR*>*i* could prove ambiguous.

For example, the cash flows of – 800, +1720 and –924 at the end of years zero, one and two respectively, produce a net present value of zero when discounted at both 5% and 10% per annum:

$$\begin{cases} -800+\dfrac{1720}{1.05}-\dfrac{924}{(1.05)^2}=0 \\ -800+\dfrac{1720}{1.10}-\dfrac{924}{(1.10)^2}=0 \end{cases}$$

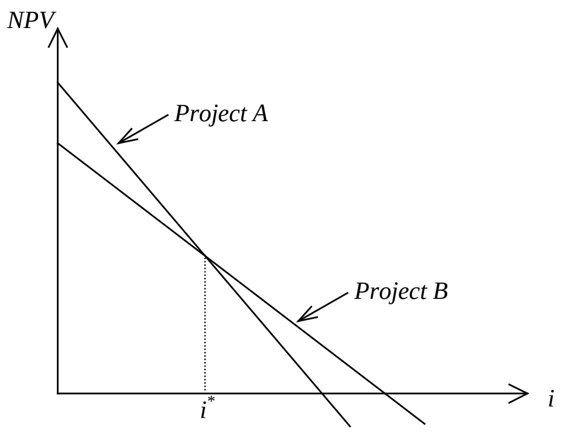
so that 5% and 10% are both *IRR*.

Therefore, two changes in the signs of the cash flows, from negative to positive to negative, result in two internal rates of return. Similarly for *n* changes in the signs of the cash flows there may be up to *n* solutions to the *IRR*.

Furthermore, some sets of solutions may have both positive and negative values. A conceptually difficult problem is that there may be *no real number* to represent the *IRR*.

A more serious problem arising from the use of the *IRR* arises in the *ranking of projects*. It will not always be the case that *the project with the higher IRR should be preferred*: consider two projects *A* and *B*, whose different profiles of costs and benefits are represented below.

Figure 5.2



Project *B* has the higher *IRR* and would always be chosen if the criterion *IRR*>*i* is used, but at any rate of discount lower than i^* (the rate that identifies the '*switching-value*' of the projects). Project *A* has a higher *NPV* and would therefore maximize the return on the funds employed. For these reasons, *the calculation of NPV is to be preferred* as the basis for the appraisal of projects.

Since the overall objective is to maximize investors' wealth (project's total net wealth), the criterion should be based on the *NPV* method which measures changes in *absolute wealth*, and not the internal rate of return approach which measures *relative wealth*.

Needless to say, in case of ranking of projects, comparisons of projects must be based on the same grounds, for example, (1) the same length of investment periods (duration) (2) the same costs (budgets), (3) measure benefits in the per-unit of cost (i.e. benefit-cost ratio)

5.6 Inflation in Project Appraisal

It is absolutely essential to consider the effects of inflation on the relevant cash flows of a project, otherwise the cash flows may be incorrectly evaluated. In practice, rates of inflation are unlikely to be known with any certainty. Nevertheless, we shall assume that we are given predetermined rates of inflation. How may inflation best be incorporated into investment appraisal?

Recall the relationship between the money (nominal) rate of interest, denoted *m*, and the real rate of interest *r*, when *i* is the general rate of inflation, is given by:

$$(1+m) = (1+i)(1+r) \tag{5.10}$$

Discounting the money cash flows

The net present value is found in the usual way by discounting the actual money cash flows for each year *t*, denoted M_t, at the actual money market rate of interest:

$$NPV = M_0 + \frac{M_1}{1+m} + \frac{M_2}{(1+m)^2} + \frac{M_3}{(1+m)^3} + \cdots \tag{5.11}$$

The actual *cash inflows from sales* depends on the rate at which selling prices are increasing. *Cash outflows for labor costs* reflect changes in *wage rates*, and *materials costs* and *overheads* rise according to their specific rates of cost inflation.

Discounting the real cash flows

A real cash flow can be derived by taking a money cash flow and converting it into its current purchasing power equivalent. The real cash flow in t years' time, R_t is equal to the money cash flow in t years' time, discounted for t years at the general rate of inflation.

$$R_t = M_t \big/ (1+i)^t \qquad (5.12)$$

So, (5.11) can be rewritten as

$$NPV = M_0 + \frac{M_1}{(1+r)(1+i)} + \frac{M_2}{(1+r)^2(1+i)^2} + \frac{M_3}{(1+r)^3(1+i)^3} + \cdots$$

From (5.12),

$$NPV = R_0 + \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \frac{R_3}{(1+r)^3} + \cdots \qquad (5.13)$$

The net present value is found by discounting the real cash flows R_t at the real rate of interest, r.

Notice that equation (5.13) is equivalent to equation (5.11) as long as we know exact values of i, r, m, M_t and R_t. In other words, if we know that the future flows of cash receivable from sales and cash payable for labor, materials, overheads and capital expenditure are all inflated at the same rate of inflation as the general rate applicable to the discount rate, then the effects of specific and general inflation are neutralized (this is not always the case though).

Unfortunately the general rate of inflation relevant to the discount rate is unlikely to be the same as the specific rates of inflation with respect to the cash flows. Not only do materials costs, selling prices, and wage rates not rise all rise at the same annual rate, but if we use the investors' marginal rate of time preference as a discount rate then the relevant rate of inflation for the discount rate is the general rate of inflation as it affects shareholders.

This in turn reduces the purchasing power of the investors' future dividends, payable from the project. The decline in the general purchasing power of cash receivable in the hands of investors depends on (i) the basket of goods bought by shareholders and (ii) the prices of the goods in that basket.

The general change in shareholder prices may be approximated by changes in the consumer price index, and this may or may not be the same as the specific inflation rates affecting the cash flows of a particular firm.

5.7 Political Economy of Cost and Benefit*

We develop a model of the public choice mechanisms comprising a representative legislature in order to show the political sources that systematically bias public decisions toward larger than efficient projects in the area of distributive policymaking. By distributive policies we mean those projects, programs, and grants that concentrate the benefits in geographically specific constituencies, while spreading their costs across all constituencies through generalized taxation.

The model reveals three important sources of bias. The first is a consequence of the political definition of benefits and costs and its divergence, in important respects, from the economic definition. The second source stems from the districting mechanism which divides the economy into n disjoint political units called districts. The method of project financing through generalized taxation constitutes the third source of bias. Moreover, we show that the mechanism of popular election of legislative representatives complements these sources of bias so that these three sources, in conjunction with the reelection mechanism, explain the inefficiency of political choice.

The General Approach

A distributive policy is a political decision that concentrates benefits in a specific geographic constituency and finances expenditures through generalized taxation. While it is clear that all policies have a geographic incidence of benefits and costs, what distinguishes a distributive policy is that benefits are geographically targeted. In contrast, a nondistributive program through having geographic incidences, is fashioned with nongeographic constituencies in mind, for example, socioeconomic groups.

A distributive policy for the j-th district, $P_j(x)$, is a project located in that district, where x is a decision parameter. Although x may be treated as a vector of project characteristics, we assume that x simply describes the scale or size of the project. Let $b(x)$ represent the present value of the economic benefits which includes consumption benefits and potential pecuniary gains to producers.

Assumption 1: $b'(x) > 0,\ b''(x) < 0$

* This section is drawn heavily from

Let $c(x)$ represent the total resource cost involved in producing the project. It decomposes into three components; $c(x) = c_1(x) + c_2(x) + c_3(x)$, where $c_1(x)$ is the real resource expenditures for project inputs spent in the constituency in which the project is located; $c_2(x)$ is the real resource expenditures for project inputs spent outside the district; $c_3(x)$ is non-expenditure real resource costs imposed on the district (e.g. nonpecuniary expenditures, such as the destruction of the natural environment, and pecuniary externalities in the form of price rises to consumers in factor markets).

Assumption 2: $c_i'(x) > 0,\ c_i''(x) \geq 0,\ i = 1,2,3$

The expenditures are financed through taxes so that the tax bill for $P_j(x)$ is

$$T(x) = c_1(x) + c_2(x) \tag{5.14}$$

Assume that a tax system that covers all expenditures, assigning nonnegative tax share t_i to the i-th district, where $\sum_{i=1}^{n} t_i = 1$ and n is the number of districts.

The tax bill for the i-th district for the project $P_j(x)$ is $t_i\left[c_1(x) + c_2(x)\right]$. Let us explore the efficiency criterion now.

Model (1): ***Maximizing Economic Efficiency***

This model requires the maximization of economic net benefits. This is given simply by

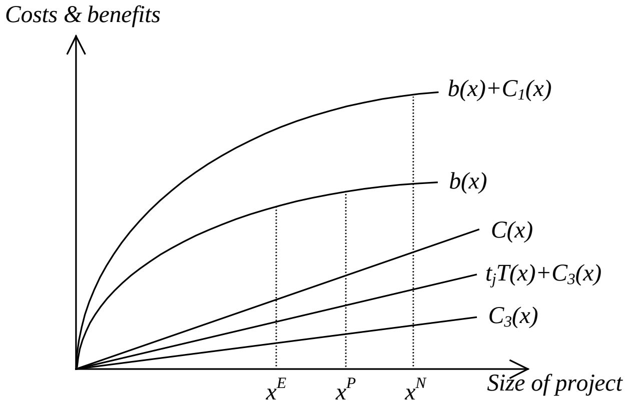
$$\max_x E(x) = b(x) - c(x) \tag{5.15}$$

The first and second order conditions are

$$b' - c' = 0 \quad \text{and} \quad b'' - c'' < 0 \tag{5.16}$$

The second-order condition follows directly from assumption 1 and assumption 2 so that the solution to (5.15) x^E in Figure 5.3 is a unique global maximum.

Figure 5.3 Costs and Benefit under Different Size



The first transformation of the standard approach is the politicization of economic costs. That is, a valuation of a project may come in either of two forms: a pecuniary gain to a factor owner or a benefit to a project consumer. Partly as a consequence of these distributional effects, and partly for additional reasons enumerated below, the political evaluation of pecuniary effects diverges from their economic treatment.

We may distinguish several classes of agents who are differentially advantaged or harmed by the provision of $P_j(x)$; (1) *in-district consumers*, who receive benefits through consumption of the public project but are unaffected by pecuniary effects; (2) *in-district factor owners*, who obtain pecuniary gains in production of, as well as benefits in consumption from $P_j(x)$; (3) *out-of-district factor owners*, who obtain pecuniary gains in production (but no consumption benefits since they do not reside in the local constituency); (4) *in-district consumers who make factor market purchases*, who obtain consumption benefits from the project but suffer pecuniary losses in the form of higher prices for factors; (5) *out-of-district purchasers of factors*, who suffer pecuniary losses through higher prices in factor markets (and who obtain no consumption benefits since they do not reside the district).

Model (2): ***A Single Political Constituency***

Consider the case of a single national constituency. Then $t_j = 1$ and expenditures earnmarked for the constituency now consist of all expenditures, $c_1(x) + c_2(x)$. With these assumptions, the political objective function reduces to

$$\begin{aligned}\max_x P(x) &= [b(x) + c_1(x) + c_2(x)] - [c_1(x) + c_2(x) + c_3(x)] \\ &= b(x) - c_3(x)\end{aligned} \tag{5.17}$$

The first-order and the second-order conditions are

$$b'-c'_3=0 \text{ and } b''-c''_3<0 \tag{5.18}$$

The solution to equation (5.18) yields point x^P on Figure 5.3 above.

Model (3): ***Every District for Itself***

Let us decompose the single national constituency into multiple, disjoint political units, *districts* with representation in a legislature. Each district, through its representative, is presumed to maximize its net (private) benefits without regard to the costs imposed on other districts. Publicly supported projects are funded through taxes which fall primarily on other districts. Hence the benefits are concentrated while the costs are diffused. Consider legislator *j's* maximand regarding his district's project:

$$N_j(x)=b(x)+c_1(x)-t_jT(x)-c_3(x) \tag{5.19}$$

The legislator is presumed to maximize the district's private benefits, $b(x)+c_1(x)$, minus its share of the taxes, $t_jT(x)=t_j\left[c_1(x)+c_2(x)\right]$ minus the externalities of the project which fall on the district, $c_3(x)$.

Maximization of (5.19) yields the first-order condition

$$\begin{aligned} &b'+c_1'-t_jT'-c_3'=0 \\ \Rightarrow\ &b'+c_1'=t_j(c_1'+c_2')+c_3' \end{aligned} \tag{5.20}$$

The second-order condition requires

$$b''+c_1''-t_j(c_1''+c_2'')-c_3''<0$$

Equation (5.20) has a solution yielding a maximum point x^N for (5.19) depicted in Figure 5.3.

Suppose $t_j=t_j(z)$ where *z* might be any characteristic. Then the following comparative statics result may be established.

***Theorem*:** Let x^N be the optimum for (5.19). Then $dx^N/\partial z>0$ if and only if $t_j'<0$.

Proof: Rewriting (5.19), we have

$$N(x) = b(x) + [1 - t_j(z)]c_1(x) - t_j(z)c_2(x) - c_3(x),$$

and first-order condition

$$N'(x) = b'(x) + [1 - t_j(z)]c_1'(x) - t_j(z)c_2'(x) - c_3'(x) = 0.$$

The second-order condition is

$$N''(x) = b''(x) + [1 - t_j(z)]c_1''(x) - t_j(z)c_2''(x) - c_3''(x) < 0.$$

Totally differentiating the first-order condition yields

$$\{b''(x) + [1 - t_j(z)]c_1''(x) - t_j(z)c_2''(x) - c_3''(x)\}dx = [c_1'(x) + c_2'(x)]t_j'(z)dz.$$

Thus,

$$\frac{dx}{\partial z} = \frac{[c_1'(x) + c_2'(x)]}{\{b''(x) + [1 - t_j(z)]c_1''(x) - t_j(z)c_2''(x) - c_3''(x)\}} t_j'(z).$$

The numerator of the coefficient on the right hand side is positive and the denominator is negative. The coefficient is thus negative, establishing that the sign of $dx/\partial z$ is opposite that of $t_j'(z)$:

$$t_j'(z) < 0 \rightarrow \frac{\partial x}{\partial z} > 0$$

$$t_j'(z) > 0 \rightarrow \frac{\partial x}{\partial z} < 0$$

Corollary: If district tax share is a declining function of the number of districts (*n*), then the degree of inefficiency in project scale $(x^N - x^E)$ is an increasing function of the number of districts.

Proof**:** Let $t_j = t_j(n)$ with $t_j'(n) < 0$. Then from the above theorem, $\partial x^N / \partial n > 0$.

The corollary indicates that when taxes are apportioned as a decreasing function of the number of political units – for example, $t_j(n) = 1/n$ for all *j* – then the optimum project scale for any district grows as the policy is more finely partitioned into districts. However, the above theorem is more general, for it applies to tax mechanisms that may be the function of any politically relevant characteristic. If a district's tax share is a decreasing function of certain characteristics of its legislator (*e.g.* an important committee chairman), then we can associate increasingly inefficient projects with particular kinds of districts as defined by relevant characteristics. In all cases, the equilibrium scale of a district's project changes with respect to some tax criterion in precisely the opposite way the tax share changes with respect to that criterion.

This model concludes that since political institutions fundamentally alter the perceptions and incidence of benefits and costs, they systematically bias project choices away from the efficient outcomes. In the context of distributive politics, this was shown to imply larger projects and programs than are economically warranted.

Appendix 1 Real Option Approach to Investment

Introduction

Recall applied analyses of neoclassical investment model generally fall into two groups:

1) The user cost of capital approach of Dale .W. Jorgenson

   Treating capital investment as a purchase of a durable good, Jorgenson defines the user cost of capital to be the rental cost of the capital (determined by the purchase price, opportunity cost of funds, depreciation rates and taxes). Firm's desired stocks of capital are determined by the equality of the value of the marginal product of capital and the user cost of capital.

2) The other approach of James Tobin compares the replacement cost of a marginal investment to its capitalized value. Tobin's q, or marginal q, is the ratio of this capitalized value to the replacement cost of the investment. Tobin's q approach provides a simple rule to guide investment: If $q>1$, the firm should invest, and if $q<1$, the firm should not invest and should shrink its existing capital stock. The firm's equilibrium capital stock is achieved when $q=1$.

Both variants of the neoclassical model rely on *the net present value (NPV) rule*: A firm should undertake investment projects with positive *NPV*. They make two subtle assumptions as well. (1) *invested capital can be sold easily to other users* (i.e. it is reversible), (2) *each investment opportunity facing the firm is a once-and-for-all opportunity* (if the firm declines the project, it will never have the choice to reconsider).

The starting point for *the new approach (real option) to investment of Dixit and Pindyck* (1994) is that many real-world investment decisions violate these subtle assumptions, and *irreversibility and a chance for delay are important considerations*.

This importance reflects the observation that *the possibility of delay gives rise to a call option*: the firm has the right, though not the obligation, to buy an asset (the investment project) at some future time at its discretion.

To the extent that investment is *irreversible* – a feature of the new view models – making an investment extinguished the value of the call option, or "*real option*". *The value of the lost option is a component of the opportunity cost of investment*.

The illustrations of irreversibility (or sunk costs) are not hard to provide in many cases, including *firm-specific marketing costs* and even less specific capital such as *computers or general use machinery are irreversible if secondary markets are inefficient*.

A value to delay is also plausible on *a priori* grounds. While delay entails possible costs, it confers *benefits in the form of new information about the project's value during the period of* *delay*.

A Very Brief Review of Option

The simplest financial option, *a European call option*, is a *contract with* the following conditions: (c.f. American option)

1) At a prescribed time in the future, known as *the expiry date* or *expiration date,* the holder of the option may
2) purchase a prescribed asset, known as *the underlying asset* or, briefly, the *underlying*, for a
3) prescribed amount, known as the *exercise price* or *strike price*.

The word "*may*" in this description implies that for the holder of the option, this contract is a *right* and not an *obligation*.

The other party to the contract, who is known as *the writer*, does have *a potential obligation*: he *must* sell the asset if the holder chooses to buy it. Since the option confers on its holder *a right with no obligation* it has *some value*. Moreover, it must be *paid for at the time of opening the contract*. Conversely, the writer of the option must be *compensated for the obligation* he has assumed.

Two main concerns are

1) how much would one *pay for this right*, i.e. what is *the value of an option*?
2) how can the writer *minimize the risk associated with his obligation*?

A Call Option

How much is the following option now worth? Today's date is 25 June, 199x.

1) On 22 December199x the holder of the option may,
2) purchase one XYZ share for ¥250

In order to gain an intuitive feel for the price of this option, let us imagine two possible situations that might occur on the expiry date, 22 December 199x, nearly six months in the future.

If the XYZ share price is *¥270* on 22 December 199x, then the holder of the option would be able to purchase the asset for only *¥250*. This action, which is called *exercising the option*, yields an immediate profit of *¥20*. That is, he can buy the share for ¥250 and immediately sell it in the market for ¥270.

¥270 - ¥250 = *¥20 profit*

On the other hand, if the XYZ share price is only ¥230 on 22 December 199x, then *it would* *not be sensible to exercise the option*.

If the XYZ share only takes the value ¥230 or ¥270 on 22 December 199x with equal probability (1/2), then the expected to be made is

$$\frac{1}{2}\times 0+\frac{1}{2}\times 20=¥10$$

Ignoring interest rates for the moment, it seems reasonable that the order of magnitude for *the value of the option is ¥10*.

Of course, valuing an option is not as simple as this, but let us suppose that the holder did indeed pay ¥10 for this option. Now if the share price rises to ¥270 at expiry he has made a net profit calculated as follows:

Profit on exercise	¥20
Cost of option	¥10
Net profit	¥10

The downside of this speculation is that if the share price is less than ¥250 at expiry he ahs lost all of the ¥10 invested in the option, giving *a loss of 100%.*

If the investor purchases the share for ¥250 on 25 June 199x, the corresponding profit or loss of ¥20 would be only $\pm 8\%$ of the original investment. Option prices thus respond in an exaggerated way to changes in the underlying asset price. This effect is called *gearing*.

It seems reasonable that the higher the share price is now then the higher the price is likely to be in the future. Thus *the value of a call option today depends on today's share price*. Similarly, *the dependence of the call option value on the exercise price* is obvious: *the lower the exercise price, the less that has to be paid on exercise and so the higher the option value*.

Put Option

The option to *buy* an asset discussed above is known as a *call option*. The right to *sell* an asset is known as a *put option* and has payoff properties, which are opposite to those of a call. A put option allows its holders to sell the asset on a certain date for a prescribed amount. The writer is then obliged to buy the asset. Whereas the holder of a call option wants the asset price to rise – the higher the asset price at expiry the greater the profit – the holder of a put option wants the asset price to fall as low as possible. The value of a put option also increases with the exercise price, since with a higher exercise price move is received for the asset at expiry.

A Simple Example of Real Option

The implications of irreversibility and the option-like nature of an investment opportunity can be demonstrated most easily with a simple two period example.

Consider a firm's decision to invest irreversibly in a widget factory. The factory can be build instantly at a *cost I* and will produce one widget per year forever with *zero operating cost* (for simplicity). Currently the price of widgets is ¥100, but next year the price will change. With probability *q*, it will rise to *¥150* and with probability (*1-q*), it will fall to *¥50*. *The price will then remain at this new level forever*.

$t = 0$ $\qquad$ $t = 1$ $\qquad$ $t = 2$

$$P_0 = ¥100 \begin{cases} \xrightarrow{q} P_1 = ¥150 \rightarrow P_2 = ¥150 \rightarrow \cdots \\ \xrightarrow{1-q} P_1 = ¥50 \rightarrow P_2 = ¥50 \rightarrow \cdots \end{cases}$$

We assume that this risk is fully diversifiable, so that the firm can discount future cash flows using *the risk-free rate* (*say 10%*).

For the time being we will set *I*=¥800 and *q*=0.5. Given these values for *I* and *q*, is this a good investment? Should we invest now or wait a year and see whether the price goes up or down?

Suppose we invest now. Calculating the net present value of this investment in the standard way, we get

$$NPV = -800 + \sum_{t=0}^{\infty} 100/(1.1)^t = -800 + 1100 = ¥300 > 0$$

The *NPV* is positive, hence it would seem that we should go ahead with this investment.

This conclusion is incorrect because the calculations above ignore *a cost – the opportunity cost of investing now*, rather than waiting and keeping open the possibility of not investing.

To see this, calculate the *NPV* of this investment opportunity, assuming instead that we wait one year and then invest only if the price goes up:

$$NPV = 0.5 \times \left[-800/1.1 + \sum_{t=1}^{\infty} 150/(1.1)^t \right] = ¥386$$

(Note that in year 0, there is no expenditure and no revenue. In year 1, the ¥800 is spent only if the price rises to ¥150 which will happen with probability 0.5.)

The NPV today is higher if we plan to wait a year, so clearly *waiting is better than investing now*.

Note that *if our only choice is to invest today or never invest, we would invest today*. In that case there is no option to wait a year, and hence no opportunity cost to killing such an option, so the standard *NPV* rule would apply. We would likewise invest today *if next year we could disinvest and recover the ¥800*. (e.g. Art market. If you don’t buy now, you will loose your opportunity to buy the specifit painting (any other art objects) almost forever.)

Two things are needed to introduce an opportunity cost into the *NPV* calculation – 1) *irreversibility* and 2) *the ability to invest in the future as an alternative to investing today*.

There are, of course, situations in which a firm cannot wait, or cannot wait very long, to invest (one example is *the anticipated entry of a competitor into a market* that is only large enough for one firm. Another example is *a patent or mineral resource lease that is about to expire*).

The less time there is to delay, and the greater the cost of delaying, the less will irreversibility affect the investment decision.

How much is it worth to have the flexibility to make the investment decision next year, rather than having to invest either now or never? (We know that having this flexibility is of some value, because we would prefer to wait rather than invest now.)

The value of this flexibility option is easy to calculate; it is just the difference between the two *NPV*, that is ¥386-¥300=¥86.

Analogy to Financial Option

As we saw, *our investment opportunity is analogous to a call option on a common stock*. It gives us *the right* (which we need not exercise) to make an investment expenditure (the exercise price of the option) and receive a project (a share of stock) the value of which fluctuates stochastically.

In the case of our example, next year if the price rises to ¥150, we exercise our option by paying ¥800 and receive an asset that will be worth $V_1 = ¥1650 \left(= \sum_0^{\infty} 150 \big/ (1.1)^t \right)$. If the price falls to ¥50, this asset will be worth only ¥550, so we will not exercise the option.

The value of our investment opportunity is ¥386. It will be helpful to recalculate this value using standard option pricing methods.

Let F_0 denote the value today of the investment opportunity, that is, what we should be willing to pay today to have the option to invest in the widget factory, and let F_1 denote its value next year.

Note that F_1 is a random variable; it depends on what happens to the price of widgets. If the price rises to ¥150, then F_1 will equal $\sum_0^{\infty} 150/(1.1)^t - 800 =$ ¥850. If the price falls to ¥50, the option to invest will go unexercised, so that F_1 will equal 0. Thus we know all possible values for F_1. The problem is to find F_0, the value of the option today (=¥386).

Conclusion is the same. We should wait and keep our option alive, rather than invest today (In the case of calculating the value of the option, we base on the construction of a risk-free portfolio, which requires that one can trade (hold a long or short position in) widgets. For detail, see Pindyck (1991).

Appendix 2 Trying to Value A Life: The 55 mph speed limit decision

By lowering average road speeds by 4.8 miles per hour, the legal limit had two main effects. First, it took longer to make a journey. The costs of the regulation would therefore be bound by multiplying the number of extra hours by the value of time. Second, there would be fewer fatalities. The benefits of the regulation depend on the number of lives saved. Data on the number of hours spent on the road were available. The main problem was how to estimate the number of lives that would be saved.

Three Methods of Valuing a Life

Two methods of valuing a life are variants of the human capital approach. They measure the value of people's lives by their contribution to the economy.

Method (I) looks at the economy in terms of national income. At the individual level, a person's contribution is the present discounted value of future earnings over one's expected lifetime. Forester, *et al* (1984) show that the average person was 33.5 years old, earning the 1981 national average of $15496 and that with a retirement age of 65 years, these earnings could be expected to last 31.5 years. The total lifetime earnings ($15496 times 31.5) when discounted at the rate of 0.5% (which assumes that the expected growth in earnings will be 0.5% greater than the opportunity rate of return on capital) equals $527200. Multiplying this value of life by the 7466 lives that were expected to be saved produced the money value of the benefits of the 55 mph speed limit.

Method (II) was similar to Method (I), except that it required deducting from earnings all the consumption that people make over their lifetime. The assumption is that it is earnings less consumption that the rest of society loses when a person dies. Forester, *et al* (1984) used a 30% consumption rate derived from budget studies by the Department of Labor. This placed the value on a life equal to $369040.

The human capital approach has the advantage that it is simple to interpret and data are readily available on this basis. However, as stressed by Mishan (1975), the human capital approach looks at the effect on society and ignores the preferences of the individual whose life is at issue.

It is preferences over risky outcomes that should be the basis for making evaluations of the loss of life. Schelling (1968) consequently argued that it is a *statistical death* that one is contemplating, not *a certain death*. By considering what individuals are willing to receive as compensation for putting up with the risk of death, Schelling provided an individualistic mechanism for measuring the value of life. Schelling distinguishes situations where actual lives were at stake from those where an anonymous person's life is at stake (which is the statistical life framework). When the individual's identity is known, valuations are likely to be much higher than when applying a small risk probability to a large, impersonal aggregate of people, to obtain a life that is predicted to be lost.

Table 5A.2-1 Benefit-Cost Ratios for the 55 mph Speed Limit

Time valued at	Life valued by		
	Human Capital Method (I)	Human Capital Method (II)	Schelling's Method
Average hourly wage	0.24	0.17	0.25
Two-Thirds of average wage	0.36	0.25	0.38
One-half of average wage	0.48	0.33	0.51
Thirty percent of average wage	0.79	0.56	0.84

Source: Forester, *et al* (1984).

Note that the relative values of cost-benefit can be compared by using a common monetary unit. However, one may be able to replace the monetary metric with time as the numeraire. For example, the 55 mph speed limit made journeys take longer in order to make them safer. Time must be given up to save lives. But these lives themselves are simply periods of expected future time availability. When discounted, this expected future time is in comparable units to the time that must be given up to undertake the safety precaution.

Let us consider the 33.5 year old person who is predicted to lose his/her life. She/he has a life expectancy of 42.4 years. If it is known that one such person in society would lose his/her life is a safety precaution were not undertaken, then all of the individuals in society in the aggregate should be willing to invest up to 42.4 years in preventive action. Using this logic, we can see whether the 55 mph speed limit decision provide more time in terms of lives saved than it used up time in making people travel more slowly.

The undiscounted benefits of the 55 mph speed limit were 316,558 years of life saved (7,466 lives times the 42.5 expected years of life in the future). The cost was the extra 456,279 years that travellers had to spend on the roads. The benefit-cost ratio would therefore be 0.69. Using time as the numeraire supports the verdict of Forester *et al* (1984) who found that the 55 mph speed limit was not "cost-effective". The result here indicate it was not "time-effective" either. No matter the method of life valuation used, the legal speed limit was not a social improvement.

Exercises

1. (Davis and Pointon (1984), p.52) A company is evaluating two mutually exclusive projects A and B, each having a useful life of three years. Project A requires an immediate capital outlay of $1,200,000 while project B requires $1,000,000. In the absence of cost and price inflation, the cash flows shown in Table 5.1 (below) would remain constant throughout the life of each project and arise at the end of each of the next three years.

Table 5.1 Estimated cash flows receivable at the end of each year in the life of each project

A		B	
Cash inflow from sales	$1,000,000	Cash inflow from sales	$800,000
Cash outflows		Cash outflows	
Labor	$200,000	Labor	$100,000
Materials	$200,000	Materials	$50,000
Other	$20,000	Other	$150,000
	$420,000		$300,000
Total	$580,000	Total	$500,000

However, it is now predicted that sales prices will increase by 10% per annum, labor costs by 20%, and material costs by 8%. Table 5.2 shows the impact of inflation on 'other costs' mentioned in Table 5.1.

Table 5.2 Money cash flows of other costs

	Year 1	Year 2	Year 3
Project A	21,000	44,000	88,720
Project B	158,000	206,000	265,000

The money cost of capital is estimated to be 15% per annum.

Question 1. Using the net present value method, determine which project is financially more attractive.

Question 2. If the consumer price index is expected to increase by 10% per annum, calculate the cost of capital in real terms.

Question 3. Describe how your result from Question 2 may be applied to the appraisal of a capital project.

2. Net Present Value of net benefit of the project is defined as follows,

$$NPV = \sum_{t=0}^{T} \frac{B_t}{(1+r)^t}$$

where B_t = net benefit, T is the life of the project (T=5).

Consider the hypothetical net flows of benefit, this project required periodic reinvestment costs.

B = -815, 900, -100, 1200, -1200

Question 1. Calculate internal rate of return.

Question 2. Under what condition, the NPV may exhibit anomalous behavior?

Question 3. If anomalous behavior is present, how do you determine appropriate discount rate? Or can you use different discount rates for borrowing and reinvestment? Discuss.

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