Chapter 1 A Framework of Taxation

1.1 Economic Impacts of Taxation[1]

Good tax system for any economy must satisfy the following properties;

(1) simplicity of administration and increases in tax compliance
(2) improve economic efficiency and reduce dead-weight loss
(3) promote long-run economic growth
(4) maintain flexibility of tax system
(5) honor society's norms of fairness and equity

Simplicity of administration refers to the ability of a department of revenue to collect the taxes due easily and economically, at the smallest cost of tax compliance refers to the taxpayers' ability to understand the tax code and pay the taxes owed with minimal effort, record keeping and costs.

Second and third properties are concerned with the efficiency issued; the second to static efficiency and the third to dynamic efficiency. As to the static efficiency, taxes distort markets by driving a wedge between the prices faced by buyers and sellers, thereby generating dead-weigh efficiency losses. The goal of tax design is to minimize the dead-weight efficiency losses for any given amount of revenues collected.

The dynamic efficiency problem is that taxes may reduce incentives to save and invest, to the detriment of long-run economic growth. The goal is to maintain incentives for saving and investment to the fullest extent possible. The fourth property, flexibility objective, is associated with the macroeconomic stabilization goal of smoothing the business cycle taxes cue the main instrument of fiscal policy. The fifth property of fairness and equity is a reminder that taxes must be consistent with society's norms in its quest for consequences as well as process equity.

1.2 Tax Incidence[2]

Who pays taxes? A first answer consists in accepting that it is the (legal) person who signs the check.[3]

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[1] This section draws heavily on Tresch (2002, Chap.11, pp.332-3)
[2] This part draws from Salanié (2003, Chap1, pp.15-34).
[3] This is often called the flypaper theory of incidence: taxes stick where they first come.

The theory of tax incidence aims at characterizing the effect on economic equilibrium of a change in taxes. The changes in prices are a target variable of the theory; ideally (if it were easy to evaluate changes in utilities) the theory should also compare the utilities of all agents before and after the tax change, so as to give a satisfactory answer to this seemingly simple question: How is the tax burden shared among the economic agents?

This section studies the real incidence of taxes both in partial equilibrium and in general equilibrium. This issue emerged in partial equilibrium as early as the seventeenth century. Both Adam Smith and David Ricardo discussed tax incidence in detail, but their whole analysis was based on supply, since they lacked an adequate concept of demand[4]. The modern analysis of partial equilibrium incidence arrived with the marginalists; however, general equilibrium effects then were relegated backstage. The theory of tax incidence in general equilibrium only emerged with Harberger (1962), which we study in detail later in this chapter.

Partial Equilibrium

The Effect of Payroll Taxes

Let us look at the effect of payroll taxes on the labor market. In most countries, social security (which finances pensions in the United States, but also unemployment and health benefits elsewhere) is financed in large part from payroll taxes based on wages. Some of these taxes are "paid" by employers and some by workers. This legal distinction is artificial: the only concepts of wages that matter are that paid by the employer (the gross wage) and that received by the employee (the net wage). Whether the employer pays 80 percent or 50 percent or 20 percent of payroll taxes is immaterial to the equilibrium gross and net wages and to the determination of employment.

First consider the labor market for a category of workers sufficiently skilled that the market clears in the long run. Without payroll taxes, equilibrium is figured by point E on Figure 1, which is the usual supply and demand graph in the (L,w) plane. Let us now introduce proportional payroll taxes at an infinitesimal rate dt, so that with net wage w, the gross wage now is $w(1+dt)$. For a fixed net wage, labor demand decreases so that the new equilibrium lies at point E', where both net wage and employment are lower than in E while the gross wage is higher. Thus the burden of payroll taxes is borne both by workers (since their net wage decreases). Once again, this analysis does not depend at all on who in practice pays the taxes: it does not matter whether it is the employers, the workers, or any combination of the two.

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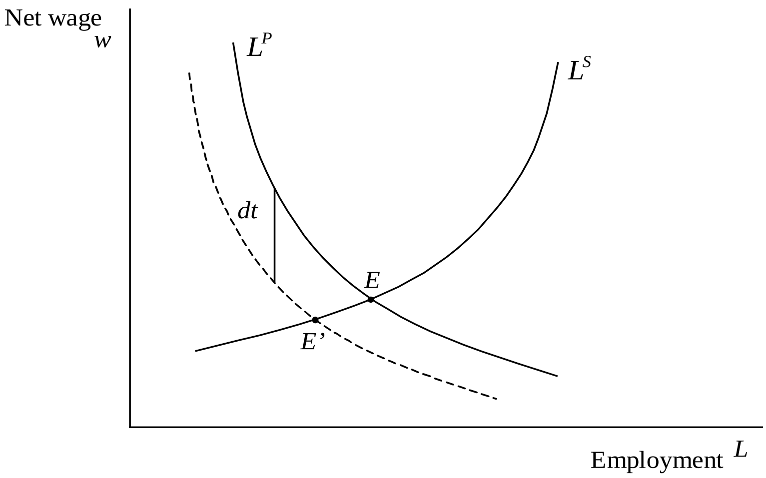
[4] Smith thought, for instance, that since workers are paid a subsistence wage, they cannot bear any of the tax burden: a tax on wages or basic consumption goods must be shifted onto the other social classes. Ricardo was the first to distinguish short-term and long-term incidence, using the Malthusian theory of labor supply adjustments.

Figure 1



The precise impact of payroll taxes obviously depends on the elasticities of the demand and supply curves, which are given (in absolute value) by

$$\varepsilon_D = -\frac{wL^{d'}}{L} \quad \text{and} \quad \varepsilon_S = -\frac{wL^{s'}}{L}$$

After a payroll tax at rate *t* is introduced the labor market equilibrium is given by

$$L^d\left(w(1+t)\right) = L^s(w)$$

To simplify, let us start from a situation where *t*=0; differentiation then gives

$$L^{d'}\left(dw + wdt\right) = L^{s'}dw$$

so that

$$-1 < \frac{\partial \log w}{\partial t} = \frac{\varepsilon_D}{\varepsilon_S + \varepsilon_D} < 0$$

Thus the net wage decreases all the more that demand is more elastic relative to supply. Similar calculations show that if we denote the gross wage $W = w(1+t)$, then

$$0 < \frac{\partial \log W}{\partial t} = \frac{\varepsilon_D}{\varepsilon_S + \varepsilon_D} < 1$$

Symmetrically the gross wage increases all the more that demand is less elastic relative to supply. Finally, the fall in employment is given by

$$-\frac{\partial \log L}{\partial L} = \varepsilon_S \frac{\partial \log w}{\partial t} = \frac{\varepsilon_S \varepsilon_D}{\varepsilon_S + \varepsilon_D}$$

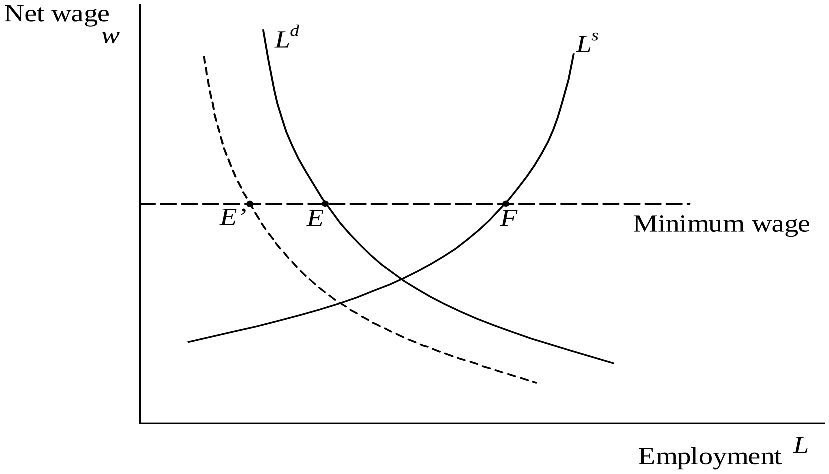
since both points *E* and *E'* lie on the labor supply curve. Since $ab/(a+b) = 1/(1/a + 1/b)$, the fall in employment is all the larger that demand and supply are more elastic.

Economists usually agree that at least for the male core of the labor market, labor supply is much less elastic than labor demand $(\varepsilon_S << \varepsilon_D)$. Then the preceding formulas show that the cost of labor hardly changes: workers bear the full burden of payroll taxes[5]. This theoretical analysis is also confirmed in empirical studies. Moreover employment moves very little since labor supply is very inelastic.

Obviously the assumption that the labor market clears may not be adequate for all akill levels. Take, for instance, a country with a minimum wage. Then let us look at the lowest skill levels (those that are affected by the existence of the minimum wage). Assume that the minimum wage is set above the market-clearing wage, as in Figure 2. Then employment is determined by demand in *E*, and there is unemployment, as measured by the distance *EF*. If payroll taxes increase, the net wage stays equal to the minimum wage since it cannot fall further, and the cost of labor increases as the payroll taxes do. Employment is set by labor demand with a higher cost of labor in *E'* and unemployment increases by the distance *E'E*. This type of analysis is why many economists in continental Europe have argued for lowering payroll taxes on the low-skilled.

[5] Whether the taxes are "paid" by employers or by workers.

Figure 2 Incidence of payroll taxes on unskilled labor



The General Analysis of Partial Equilibrium

The Competitive Case

The analysis of the incidence of the tax on a good (e.g., VAT on cars) is formally identical to that of the impact of payroll taxes on the labor market: identify the net wage to the producer price, the gross wage to the consumer price, and let the demand and supply curves now be drawn for the good under consideration. It follows that the creation (or the increase) of a VAT on cars

- increase the consumer price all the more that the demand for cars is less elastic than the supply of cars
- reduces the producer price all the more that the supply of cars is less elastic than the demand for cars
- reduces the number of cars sold in equilibrium all the more that demand and supply are more elastic.

There are two interesting special cases:

- if demand is much more elastic than supply $(\varepsilon_D >> \varepsilon_S)$, VAT hardly moves the consumer price: the producers bear the whole burden of the tax
- in the polar case where supply is much more elastic than demand $(\varepsilon_S >> \varepsilon_D)$, VAT is entirely shifted to the consumer, who bears the whole tax burden. This is called forward tax shifting[6].

The rule to remember is that the more inelastic side of the market bears the greater part of the

[6] *Backward tax shifting* refers to the case where input prices decrease to absorb at least part of the tax burden; partial equilibrium analysis by definition excludes this possibility.

tax burden.

As for the minimum wage on labor markets, one should also consider cases where regulation imposes price floors or price ceilings. For instance, many cities have laws that fix price ceilings for rents. Then an increase in taxes on rents cannot arise rents; in the long run when supply of apartments is elastic, it must result in an increase of demand rationing on the market.

The Monopoly Case

So far we looked at markets where all parties act in a perfectly competitive manner. When producers have some market power, the results may be rather different, as Cournot noticed as early as 1838. For a monopoly, for instance, profit maximization with a demand function *D* and a cost function *C* is given by

$$\max_p (pD(p) - C(D(p)))$$

which leads to the usual Lerner formula:

$$p = \frac{C'(D(p))}{1-(1/\varepsilon_D(p))}$$

where $\varepsilon_D(p) = -pD'(p)/D(p)$ is demand elasticity, assumed to be larger than one.

If we introduce a proportional tax at rate *t*, the identity of the side who “pays” the tax again does not matter. Let *p* be the consumer price; then the monopoly maximizes over *p* the profit

$$\frac{p}{1+t}D(p) - C(D(p))$$

which yields the new formula

$$\frac{p}{1+t} = \frac{C'D(p)}{1-\left(1/\varepsilon_D(p)\right)}$$

In general, this is a complex equation in *p*, so it is hard to compute the effect of the tax. In particular, it is quite possible that the consumer price increases by *more* than the amount of the tax[7], which cannot happen on a competitive market.

Assume, for simplicity, that marginal costs are constant in *c*; then the competitive supply is

[7] The reader can check this by assuming constant marginal tax and a demand elasticity that decreases in price.

infinitely elastic, and one would expect the tax to shift fully onto consumers. It is indeed the case when demand has constant elasticity, since then the right-hand sides of both Lerner formulas coincide[8]. On the other hand, if demand is linear as in $D(p) = d - p$, then the demand elasticity is

$$\varepsilon_D(p) = \frac{p}{d-p}$$

One gets by substituting in Lerner's formula

$$p = \frac{1}{2}(d + c(1+t))$$

so that the semi-elasticity of price to an infinitesimal tax is

$$\frac{\partial \log p}{\partial t} = \frac{c}{d+c}$$

and both sides of the market bear some of the burden of the tax.

In the competitive case, collecting a given amount of money as a *specific* tax (in absolute value) or an *ad valorem* tax (proportional to the value of production) changes neither allocations nor incidence. But the choice is no longer irrelevant with a monopoly. First consider the competitive case; let S be the competitive supply function. Then an *ad valorem* tax t yields a producer price p given by

$$D\big(p(1+t)\big) = S(p)$$

and collects $tpS(p)$ for the government. If we replace this tax with a specific tax $\tau = tp$, the new producer price p' is given by

$$D\big(p'+tp\big) = S(p')$$

and $p'=p$ is an obvious solution. Since the specific tax collects $\tau S(p') = tpS(p)$, neither the producer price nor the government's tax revenue change.

Now consider a monopoly with inverse demand function $P(q)$ and a cost function C. In the

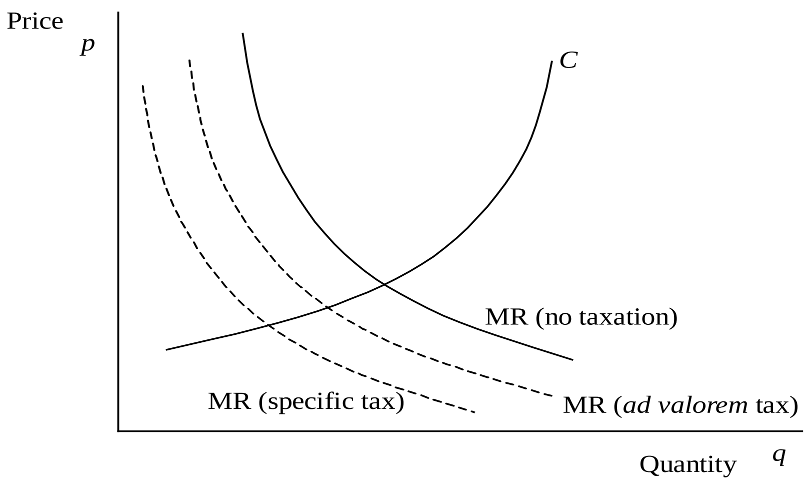
[8] Note that even then, the monopoly bears some part of the tax since that lowers its profits.

no-taxation case, the monopoly's optimum is given by

$$MR(q) = C'(q)$$

where $MR(q) = P(q) + qP'(q)$ is the marginal revenue. If the monopoly pays *an advalorem* taxat rate *t*, then marginal revenue decreases by $tMR(q)$, while a specific tax τ of course reduces marginal revenue by τ. Fix a quantity *q*. Assume that the tax parameters *t* and τ have been chosen so as to collect the same amount at production level *q*. Then $tqP(q) = \tau q$ or $\tau = tP(q)$, which implies $\tau > tMR(q)$ since marginal revenue is smaller than price. At given production and tax revenue, the specific tax thus reduces marginal revenue more than the *ad valorem* tax. It follows from Figure 3 that the quantity produced under a specific tax is lower than under an *ad valorem* tax. For a given tax revenue, an *ad valorem* reduced production less, which is good for social welfare since the monopoly already produces too little. Thus *ad valorem* taxes like VAT should be preferred to specific taxes such as some excises.

Figure 3 Taxation of a monopoly



General Equilibrium

When we studied the effect of payroll taxes on the labor market, we neglected their effects on the general price level (which feedbacks into labor demand through the production price and into labor supply through the consumer price), but also the possibility of substituting capital for labor (since the cost of capital was exogenous). Our analysis of VAT implicitly set aside the impact of an increase in VAT on incomes and therefore on demand for the good, and also its impact on wages and thus on supply. Moreover we let the money collected sink into a black hole, while in real life it is used to finance public goods or to pay various forms of income. Taking these various effects into account brings us into the world of general equilibrium theory. The founding model in the general equilibrium theory of tax incidence is that of Harberger (1962).

We consider here an economy that produces two goods *X* and *Y* from two inputs: labor *L* and capital *K*. The technologies have constant returns to scale. The total supply of either input is fixed[9], but each factor is perfectly mobile across the two sectors. The two goods are consumed by workers, capitalists, and government. To simplify the analysis, we assume that the demand functions for goods only depend on relative prices and on the gross domestic product of the economy. Thus we neglect the impact on demands of the distribution of income. This could be "justified" by assuming that all agents have identical, homothetic preferences[10]. More realistic analyses take income distribution into account; we will here stick to the assumption of identical homothetic preferences for simplicity.

The No-Taxation Economy

First assume all taxes away. Let $C_X(r,w,X)$ and $C_Y(r,w,Y)$ denote the cost functions in both sectors, where *r* and *w* are the prices of capital and labor. As returns are constant, both cost functions are proportional to production levels:

$$\begin{cases} C_X(r,w,X) = c_X(r,w)X \\ C_Y(r,w,Y) = c_Y(r,w)Y \end{cases}$$

and the prices of the goods are given by[11]

$$\begin{cases} p_X = c_X(r,w) \\ p_Y = c_Y(r,w) \end{cases}$$

Factor demands are the derivatives of cost functions with respect to factor prices; thus the demand for labor in sector *X* is

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9 Thus we neglect the influence of prices on the supplies of production factors in the economy. Letting real wages influence labor supply would hardly affect the analysis. Endogenizing the supply of capital is more difficult.

10 A preference preorder is homothetic if and only if for all x and y and all positive real numbers λ, $x \sim y$ implies $\lambda x \sim \lambda y$. It is easily seen that with such preferences the demand for each good is proportional to income (the Engel curves are lines that go through the origin). In an economy in which all agents have identical homothetic preferences, an income transfer from one agent to another leaves total demand functions unchanged.

11 This is just the factor price frontier, which states that profits per unit of production

$$L_X = c_{Xw}(r,w)X$$

where c_{Xw} is the derivative of c_X in *w*.

Equilibrium on factor markets follows:

$$\begin{cases} c_{Xw}(r,w)X + c_{Yw}(r,w)Y = \overline{L} \\ c_{Yr}(r,w)X + c_{Yr}(r,w)Y = \overline{K} \end{cases}$$

where $\overline{K}$ and $\overline{L}$ are the exogenous factor supplies.

Finally, let $X(p_X, p_Y, R)$ and $Y(p_X, p_Y, R)$ denote *the Marshallian demand functions*; equilibrium on markets for goods is

$$\begin{cases} X(p_X, p_Y, R) = X \\ Y(p_X, p_Y, R) = Y \end{cases}$$

where *R* is total income, which also equals both GDP $(p_X X + p_Y Y)$ and total factor incomes $(w\overline{L} + r\overline{K})$.

The four equilibrium conditions and the two price equations give us six equations, and there are six unknowns: p_X, p_Y, r, w, X, and *Y*. As usual, one equation is redundant: Walras's law implies that one need only consider equilibrium in three of the four markets. Thus only relative prices can be determined, as is always the case in general equilibrium markets without money.

Introducing Taxes

Let us now introduce, under the guise of redistributive *ad valorem* taxes,

- *ad valorem* taxes on factor prices in both sectors: t_{KX}, t_{KY}, t_{LX}, and t_{LY}
- *ad valorem* taxes on goods: t_X and t_Y.

The taxes on goods could represent VAT, an excise like a gasoline tax or a tax on tobacco, or a *sales tax* as in the United States. Such taxes are usually levied at different rates on different goods. The taxes on labor could be social contributions (a payroll tax) and could be reduced in some sectors for stimulation purposes. The taxes on capital classically resemble the corporate income tax, which does not touch some sectors such as agriculture or housing, but one could also think of other capital taxes.

Let us denote p_X, p_Y the producer prices, and *r* and *w* the net-of-tax factor prices. The (producer) price equations then become

$$\begin{cases} p_X = c_X\left(r(1+t_{KX}), w(1+t_{LX})\right) \\ p_Y = c_Y\left(r(1+t_{KY}), w(1+t_{LY})\right) \end{cases}$$

The equilibrium conditions on factor markets now are

$$\begin{cases} c_{Xw}\left(r(1+t_{KX}), w(1+t_{LX})\right)X + c_{Yw}\left(r(1+t_{KY}), w(1+t_{LY})\right)Y = \overline{L} \\ c_{Xr}\left(r(1+t_{KX}), w(1+t_{LX})\right)X + c_{Yr}\left(r(1+t_{KY}), w(1+t_{LY})\right)Y = \overline{K} \end{cases}$$

while equilibrium on goods markets can be written

$$\begin{cases} X\left(p_x(1+t_X), p_Y(1+t_Y), R\right) = X \\ Y\left(p_x(1+t_X), p_Y(1+t_Y), R\right) = Y \end{cases}$$

Here R is the new value of total income; it still equals GDP but now includes the tax revenue. The tax revenue T is

$$p_X(1+t_X)X + p_Y(1+t_Y)Y = w\overline{L} + r\overline{K} + T$$

where

$$T = rt_{kx}K_X + rt_{KY}K_Y + wt_{LX}L_X + wt_{LY}L_Y + p_X t_X X + p_Y t_Y Y$$

This system of equations in general does not have a closed-form solution. On the other hand, it can be solved numerically so as to study the changes in prices and quantities and the incidence of one of the taxes above in general equilibrium. This approach underlies the *computable general equilibrium*, or CGE models developed after Shoven and Whalley (1972)[12]. One can also linearize the system around the existing tax system so as to study infinitesimal changes in taxes, as in Ballentine and Eris (1975). This leads to complex calculations and to conclusions that are hard to interpret, so most of the literature focused on the effect of introducing infinitesimal taxes in a world originally without taxes. The obvious problem with this approach is that it can only be illustrative. Given the level of taxes in actual economies, nonlinearities can hardly be neglected. Thus any study that aims at realism must use computer simulations.

[12] Shoven and Whalley (1984) presents a survey of CGE models.

General Remarks

As described above, the equilibrium conditions call for three remarks. First note that in equilibrium, factors must be paid the same net-of-tax rate in both sectors, since they are perfectly mobile. While this sounds rather obvious, it has important consequences: if, for instance, capital taxation increases in sector *X*, then the net return of capital must decrease in the whole economy, and not only for capital used in sector *X*. Otherwise, capitalists would withdraw all of their money from sector *X* to invest it in sector *Y*. This would reduce the return of capital in sector *Y* and increase it in sector *X* until both are equal again.

This is very much analogous to what happens in the transportation sector. Assume that two cities *A* and *B* are only connected by two roads R_1 and R_2. If the government creates a toll on road R_1, in the very short run only motorists who take that road will bear the burden. But soon some of them will turn to the other road. Thus they will congest road R_2 and reduce congestion on road R_1. This equilibrating process will last until the perceived cost of congestion on road R_2 equals the sum of the toll and the cost of congestion on road R_1. In equilibrium, the cost of the toll on R_1 is balanced by the higher cost of congestion on road R_2.

A second remark is that some combinations of taxes are perfectly equivalent. Start from an economy without taxes, and consider raising taxes on both factors at equal rates in sector X: $t_{KX} = t_{LX} = t$. Since c_X and c_Y are homogeneous of degree one in *r* and *w*, so that their derivatives are homogeneous of degree zero, the resulting system of equations is

$$\begin{cases} p_X = (1+t)c_X(r,w) \\ p_Y = c_Y(r,w) \\ c_{Xw}(r,w)X + c_{Yw}(r,w)Y = \overline{L} \\ c_{Xr}(r,w)X + c_{Yr}(r,w)Y = \overline{K} \\ X(p_X, p_Y, p_X X + p_Y Y) = X \\ Y(p_X, p_Y, p_X X + p_Y Y) = Y \end{cases}$$

Now abolish these two taxes and replace them with a tax on good *X* at the same rate *t*; the new equilibrium system is

$$\begin{cases} p'_X = c_X(r',w') \\ p'_Y = c_Y(r',w') \\ c_{Xw}(r',w')X' + c_{Yw}(r',w')Y' = \overline{L} \\ c_{Xr}(r',w')X' + c_{Yr}(r',w')Y' = \overline{K} \\ X(p'_X(1+t), p'_Y, p'_X(1+t)X' + p'_Y Y') = X' \\ Y(p'_X(1+t), p'_Y, p'_X(1+t)X' + p'_Y Y') = Y' \end{cases}$$

It is easily seen that the solution of this system is identical to that of the preceding system, substituting only p_X with $p'_X(1+t)$. Thus both tax systems are perfectly equivalent: taxing both inputs at the same rate in one sector is equivalent to taxing the output of that sector at the same rate. It also follows that a proportional and uniform income tax (which would apply the same tax rate to every input in every sector) is equivalent to a uniform VAT on all goods.

The last remark points out a consequence of the assumed inelasticity of total factor supply: a uniform tax on all uses of a factor (e.g., $t_{LX} = t_{LY}$) is entirely borne by that factor. As partial equilibrium incidence theory suggested, it reduces its net-of-tax price one for one and leaves all quantities and after-tax prices unchanged. This is easily seen by rewriting the system of equations as above.

Final Remarks

Harberger's model gives a good account of the complexity of the reaction of private agents to a tax in general equilibrium, especially through the interaction of factor substitution effects and volume effects. However, neglecting income effects certainly is a rather restrictive assumption, given the tax take in our economies. Moreover the preferences that we assumed for the agents are not realistic. These two drawbacks can be remedied by resorting to more complicated analytical or numerical computations. It is more difficult to go beyond the wholly neoclassical character of the model. This is all the more annoying when it is important to take into account the existence of other distortions in the economy (e.g., the minimum wage for the payroll tax, or imperfect competition in some sectors). The static perspective also is restrictive. If one considers taxes over the whole life cycle of an agent, then the incidence of capital taxation can be rather different, since a given agent may live on labor income when he is young and on capital income when he is old (see Fullerton and Rogers 1993). Finally, we worked in a closed economy. In practice, capital is mobile (less perfectly than is often said) across frontiers, which makes its supply more elastic and therefore must reduce the taxation burden it bears. The final note must be that while we economists are in relative agreement on the incidence of payroll taxes, such is not the case for the corporate income tax.

To conclude this section, let us mention the special case of the incidence of taxes on durable goods that are in fixed supply. The simplest example is that of land. Assume that the government creates a yearly tax proportional to the area of land owned by each taxpayer. This new tax reduces the value of land one for one, since the supply of land is assumed to be inelastic. Therefore the agents who own land when the tax is announced bear the whole burden of the tax. On the other hand, future landowners do not bear any burden, since the discounted value of the taxes they have to pay is exactly equal to the decrease in the price they pay for their purchase of land. Thus any change in the expected income flow from a durable good whose supply is fixed is entirely reflected in its price. This effect is called fiscal capitalization; it plays a very important role in the analysis of property taxes.

1.3 Distortions and Welfare Losses[13]

A traditional view of the role of the economist is that his task is to take governmental objectives as given and then to find a way to implement them that minimizes distortions, or equivalently, that reduces the efficiency of the economy by as little as possible. But what are these distortions, and how can they be measured? At a Pareto optimum the marginal rates of substitution of all consumers are equal to the technical marginal rates of substitution of all firms. Under the usual conditions and without taxation, the competitive equilibrium is a Pareto optimal since every consumer equates his marginal rates of substitution to the relative prices, while every firm equates its marginal rates of substitution to the relative prices. Once taxes are introduced in such an economy, the relative prices perceived by various agents differ: for instance, consumers perceive after-tax prices, while producers perceive before-tax prices. In these conditions equilibrium does not lead to the equality of marginal rates of substitution any more, and it cannot be a Pareto optimum. The price system does not coordinate the agents efficiently any more since it sends different signals to different agents.

To make this discussion more concrete, consider the very simple example of a two-good, one-consumer, and one-firm economy. The consumer's utility function over goods 1 and 2 is given by $U = C_1 C_2$; the firm transforms good 1 into good 2 through a production function $X_2 = X_1 / c$. We normalize the price of good 1 to one. The consumer's initial resources consist in one unit of good 1.

Without taxation, the equilibrium is easily computed. Since the technology exhibits constant returns, the price of good 2 must equal *c*, and the firm makes zero profit. So the consumer's budget constraint is

[13] This part draws from Salanié (2003, Chap2, pp.35-57).

$$C_1 + cC_2 = 1$$

Maximizing the utility gives $C_1 = 1/2$ and $C_2 = 1/2c$, and therefore the utility is $U = 1/4c$. Note that the marginal rate of substitution of the consumer is

$$\frac{\partial U / \partial C_1}{\partial U / \partial C_2} = \frac{C_2}{C_1} = \frac{1}{c}$$

which equals the technical marginal rate of substitution of good 1 for good 2. As expected, the equilibrium coincides with the single Pareto optimum of this economy.

Let us now introduce a specific tax t on good 2; the tax revenue is redistributed to the consumer as a lump-sum transfer T. Since the production function has constant returns, the supply of good 2 is *infinitely elastic* and the tax is entirely borne by the consumer, so that the consumer price of good 2 is $(c+t)$. The budget constraint becomes

$$C_1 + (c+t)C_2 = 1 + T$$

and we obtain

$$\begin{cases} C_1 = \dfrac{1+T}{2} \\ C_2 = \dfrac{1+T}{2(c+t)} \end{cases}$$

Now the marginal rate of substitution of the consumer is

$$\frac{\partial U / \partial C_1}{\partial U / \partial C_2} = \frac{C_2}{C_1} = \frac{1}{c+t}$$

This differs from the marginal rate of substitution, which is still $1/c$. The equilibrium is not a Pareto optimum any more: the tax creates a divergence between the relative prices perceived by the consumer and by the firm, which leads to an inefficient allocation of resources in the economy.

By definition, $T = tC_2$ and by substituting for T:

$$\begin{cases} C_1 = \dfrac{c+T}{2c+t} \\ C_2 = \dfrac{1}{2c+t} \end{cases}$$

which gives a utility

$$U(t) = \frac{c+t}{(2c+t)^2}$$

An elementary computation shows that

$$U - U(t) = \frac{t^2}{4c(2c+t)^2}$$

so that the utility loss (due to the fact that the equilibrium is not Pareto optimal) is a second-order term in *t*. This is called the *deadweight loss* or *excess burden* of the tax[14].

Note that the loss exists even though the government returns the proceeds of the tax to the consumer (by construction, $T = tC_2$). It is due to the fact that the producer and the consumer do not perceive the same relative prices; the increase in the relative consumer price of good 2 leads to an excessive consumption of good 1 and an inefficiently low consumption of good 2.

This example shows two points that will recur through this chapter. First, the distortions induced by taxes are channeled by the divergences between the prices perceived by the various agents. Second, the resulting welfare losses are second-order terms in the tax parameters. On the other hand, it is not a forgone conclusion that taxing a good leads to an underproduction and /or an underconsumption of that good: the substitution effect may be masked by an income effect in the other direction. For the standard consumption good, such a phenomenon is associated to a Giffen good and can therefore be considered a rarity. However, it is less implausible for labor supply and savings behavior.

We are now going to study the effects of taxes on the main economic decisions:

- effect of the income tax on labor supply
- effect of taxing interest income on savings
- effect of taxes on risk-taking.

We will then seek to quantify the deadweight losses due to taxes.

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[14] Of course, its precise measure depends on what utility function is used to represent preferences.

1.3.1 The Effects of Taxation on Labor Supply

We will focus here on the main economic decisions, those that play a central role in tax policy debates. In each case we will adopt a partial equilibrium viewpoint; for instance, we will neglect the effect of the income tax on workers' wages.

Labor Supply

Wages affect labor supply in an ambiguous way. A higher wage makes work more attractive relative to leisure (by the substitution effect), but it also increases the demand for leisure (by the income effect) if leisure is a normal good. The same effects come into play when looking at the income tax.

The Standard Model

Consider a consumer with utility function $U(C,L)$, where *C* is consumption of an aggregate good of unit price and *L* is labor (so that *U* increases in *C* and decreases in *L*). Assume that a proportional income tax at rate *t* is created so that the budget constraint becomes

$$C \leq (1-t)(wL+R) \equiv sL+M$$

where *R* represents nonlabor income (which is taxed at the same rate as labor income). We define $s=(1-t)w$ and $M=(1-t)R$.

The creation of (or an increase in) the income tax has three effects:

1. by lowering *M*, it reduces income; in the usual case where leisure is a normal good, this reduces the demand for leisure and thus increases labor supply
2. the decrease in *s* goes in the same direction since it also reduces income
3. the decrease in s (the relative price of labor), however, makes labor less attractive and thus reduces the supply of labor.

Effects 1 and 2 are *income effects*, which depend on the average tax rate, while the *substitution effect* 3 only depends on the marginal tax rate. This hardly matters here since the tax is proportional, but it may become important with a progressive income tax.

To evaluate these effects, start with

$$\frac{\partial L}{\partial t}=\frac{\partial L}{\partial s}\frac{\partial s}{\partial t}+\frac{\partial L}{\partial M}\frac{\partial M}{\partial t}$$

The Slutsky equation is

$$\frac{\partial L}{\partial s} = S + L\frac{\partial L}{\partial M}$$

where $S > 0$ is the Slutsky term, that is, the compensated derivative of labor supply with respect to the net wage:

$$S = \left(\frac{\partial L}{\partial s}\right)_U$$

This yields

$$\frac{\partial L}{\partial t} = -wS - (wL + R)\frac{\partial L}{\partial M}$$

The first term on the right-hand side is *the substitution effect* and is clearly negative. The second term comes from the *two income effects*; it is positive if leisure is a normal good, and it is multiplied by income. This suggests that the income effect is smaller for low-income individuals. Thus the income tax may have more disincentive effects on the poor than on the rich, other things equal.

One could illustrate this formula with a Cobb-Douglas utility function $U = a\log C + (1-a)\log(\bar{L} - L)$, but it is easy to see that the $(1-t)$ term in the budget constraint then only reduces utility without modifying labor supply. The income effect and the substitution effect exactly cancel out, and taxation does not change labor supply. As is often the case, the Cobb-Douglas specification is a very special one[15].

Obviously a proportional tax is a very bad approximation to the real world income taxes. It is nevertheless easy to analyze simple variants. Thus let us create a negative income tax G that is a benefit given to all individuals independently of their income[16]. Then the after-tax income becomes $(sL + M + G)$; other things equal, the presence of the *G* term adds an income effect that reduces labor supply. If the negative income tax is financed by increasing *t*, then the effects described above come into play. For poor individuals, going from a proportional income tax to a negative income tax seems to reduce labor supply unambiguously. This remark, however, neglects the fact that in most developed countries the poorest

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[15] It can be checked that if preferences are CES with an elasticity of substitution σ, then the income tax reduces labor supply if and only if $\sigma > 1$.

[16] So that the net tax paid is $t(wL + R) - G$, which may be negative.

households receive large means-tested benefits. These transfers should be modeled in order to understand the labor supply of the poor.

Criticisms and Extensions

The standard model implicitly assumes that workers can choose their hours *L* freely. In fact the number of hours worked may not be shosen so easily, especially in developed countries where working hours are regulated and part-time work may not be the result of a spontaneous choice. Thus it is interesting to look at the participation decision, that is the choice between not working and working a conventional number of hours *L*. For simplicity, let us neglect part-time work and assume that the utility function is $U = u(C) - v(L)$, with $v(0) = 0$; then we must compare $\left(u((1-t)(wL+R)) - v(L)\right)$ and $u((1-t)R)$. Note that the participation decision is determined by the average and not the marginal tax rate.

The derivative in *t* of the difference of these two utilities is

$$-(wL+R)u'((1-t)(wL+R)) + Ru'((1-t)R)$$

Thus it appears that participation decreases in *t* if and only if $xu'(x)$ is increasing[17], which seems reasonable. Progressivity further reduces the incentive to participate, since the average tax rate is higher when the individual works than when he does not. This analysis of the decision to participate also applies to the decision to retire, with the *caveat* that pension rights depend on contributions paid.

One could also reinterpret the standard model by analyzing L as an effort variable of the individual, something he does to improve the productivity of his labor input; the resulting problem is formally identical, so long as effort causes an increase in wages and is costly in utility terms. This reinterpretation is useful when studying the optimal taxation problem.

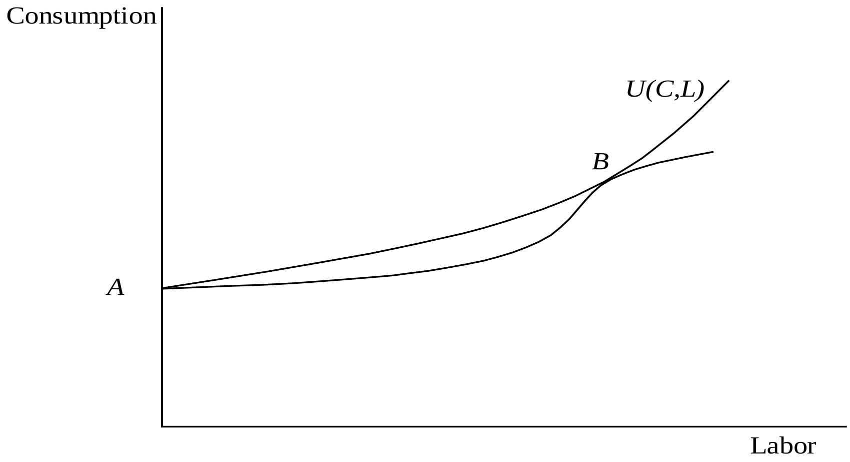
Even if we focus on labor supply, taxation impacts other variables than hours worked and effort. Consider, for instance, two jobs: job 2 is more painful and therefore better paid than job 1. Then, assuming again that utility is separable, the individual must compare the increase in utility from income $\left(u(W_2(1-t)) - u(W_1(1-t))\right)$in taking job 2 to the increase in the disutility of labor $(v_2 - v_1)$. Taxation reduces the first term and leaves the second one unchanges, which makes job 2 less attractive. Similarly taxation makes household production (tasks that are often but not always done outside of the market system, e.g., household chores and child care) more attractive, since it is not taxed.

Finally, note that real-world budget constraints are very complex and non-convex, given the actual tax-benefit systems. Figure 4 illustrates this for a rather typical developed country.

[17] Equvalently, if and only if the marginal utility of income ahs an elasticity $-xu''/u'$ that is lower than one.

The S-shape represents the fact that the marginal tax rate is high both for low incomes (where benefits are means tested and more labor income means less benefits) and for high incomes (given the progressivity of the income tax). Given such a budget constraint, small tax changes may induce large changes in labor supply for a given individual (e.g., a jump from *A* to *B* in figure 4). In practice, the complexity of actual real-world tax-benefit systems makes it necessary to resort to empirical analysis.

Figure 4 Real-world Budget Constraint



Estimates of Labor Supply

The empirical literature on the estimation of labor supply functions is very rich. So-called structural estimation procedures usually start from the standard model, where maximizing $U(C,L)$ under the budget constraint $C \leq sL + M$ and the non-negativity constraint $L \geq 0$ gives the following conditions:

$L = 0$ if

$$-\frac{U_L^{'}(M,0)}{U_C^{'}(M,0)} \geq s$$

This inequality defines an after-tax reservation wage $s_R(M)$ below which the agent refuses to work.

Otherwise, *L* is given by

$$-\frac{U_L^{'}(sL+M,L)}{U_C^{'}(sL+M,L)} = s$$

which defines a function $L^*(s,M)$.

These equations lead econometricians to specify the labor supply model as a Tobit model with a latent variable[18]

$$L^* = \alpha + \beta \log s + \gamma \log M + \varepsilon$$

(where ε is an error term) and a labor supply given by

$$L = \max(L^*, 0)$$

Of course, the wage *s* is only observed when the agent works. So the Tobit model must be estimated jointly with a wage equation that explains wages as a function of characteristics of the agents *X* and an error term *u*:

$$\log s = Xa + u$$

As such, this model is not satisfactory[19]. By assuming that the current labor supply only depends on the current wage, it cannot, for instance, account for young executives who work long hours in the hope of a promotion. The model must therefore be inserted in a lifecycle perspective. It also neglects fixed costs of participation linked to transportation costs and / or to the cost of child care, or the difficulties linked to collective choice within a household.

Finally, the model must be adapted for nonproportional taxes, which define a budget constraint

$$C \leq wL + R - T(wL + R)$$

If the marginal tax rate is increasing, then the budget constraint is still convex, as shown on Figure 5. One can then define a virtual wage $s = (1 - T')w$ and a virtual income $M = C - sL$. This way labor supply is also the solution of the program that maximizes utility under the virtual budget constraint

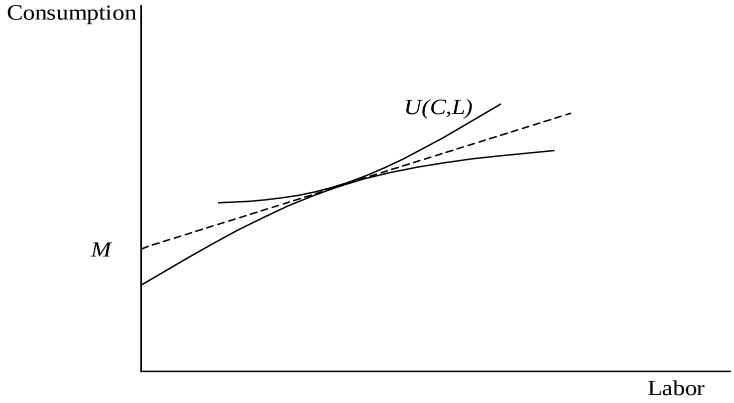
$$C \leq sL + M$$

This brings us back within the standard model, except that *s* and *M* depend on *L* through *T'* and are therefore endogenous. Then one must estimate the model with the method of instrumental

[18] The semilogarithmic specification adopted here is only an example.
[19] The selection bias must be reduced by the inverse Mill's ratio. See Heckman (1979).

variables or the maximum likelihood method.

Figure 5 Convex budget constraint



In the real world, tax systems unfortunately lead to nonconvex budget constraints such as that in Figure 4 so that the solution of the maximization program is not characterized by the first-order condition. Then one often has to discretize the choice set for labor supply L (e.g., by considering each of the values $L = 0, 1, \ldots, 60$ hours per week) and to compare the values of utility

$$U(wL + R - T(wL + R), L)$$

to find the maximum. The parameters of the utility function can then be estimated by the maximum likelihood method and used to evaluate the wage and income elasticities of labor supply[20].

Given all these complexities, it is not very surprising that estimation results vary across studies. Few doubt that leisure is a normal good[21]. The compensated wage elastictity of labor supply is more uncertain. It seems to be small for men (somewhere between 0 and 0.2, which yields an uncompensated elasticity close to zero). This is not very surprising: daily observation suggests that most men, at least in the middle part of their lives, participate in the labor force. Moreover their hours often result from collective bargaining more than from individual choice. On the other hand, the labor supply of women is much more wage elastic, especially for married women, for whom the elasticity may be between 0.5 and 1. This striking difference between men and women probably stems from the traditional sexual differentiation of roles within couples: women much more often than men choose to withdraw

[20] The reader may complete this brief tour by the survey of Blundell and MaCurdy (1999).

[21] This is confirmed, for instance, by the observation that people who inherit tend to reduce their labor supply (see Holtz, Eakin, Joulfaian and Rosen 1993).

from the labor force (or to take a part-time job) so as to raise children.

Structural estimates have been criticized because they always rely on a model that may be misspecified. Other authors have resorted to the method of *natural experiments*. This consists, in our case, in comparing the effect of a tax reform on the labor supplies of various subpopulations, some of which are more touched by the reform than others. Thus Eissa (1995) examines the effect of the Tax Reform Act of 1986 (TRA86) in the United States on the labor supply of married women. The most spectacular effect of TRA86 was the reduction of the top marginal rate of the personal income tax from 50 percent to 28 percent. Therefore the reform considerably reduced the marginal rate faced by the wives of high earners; on the other hand, its effect on wives of men with lower wages was much smaller. By comparing the changes in the labor supplies of these two subpopulations of women after TRA86, Eissa estimates that the wage elasticity of the labor supply of women in the first group is about 0.8, which falls in the same ballpark as structural estimates.

Finally, note that empirical studies sometimes use taxable income and sometimes total income. Since labor in the underground economy is by definition not taxed, one would expect that an increase in tax rates induces some agents to leave the legal sector for the underground sector of the economy, at least for part of their working hours. Lemieux, Fortin and Frechette (1994) use a survey on Québec to show that this effect is rather small on the average taxpayer, but that it matters for welfare recipients, who usually face very high marginal withdrawal rates.

1.3.2 The Effects of Taxation on Savings

In most countries, income taxation bears both on labor income and on income from savings. With perfect financial markets, taxation of labor income only changes the savings rate in that the latter depends on permanent income. We will focus here on the effect of taxation of income from savings on the time profile of consumption over the life cycle. We will therefore start by neglecting the taxation of labor income. We also assume an exogenous interest rate, which neglects general equilibrium effects.

Theoretical Analysis

Consider a consumer who lives two periods and whose labor supply is inelastic. He gets in the first period a wage w, consumes some part of it, and saves the rest according to

$$C_1 + E = w$$

In the second period, he does not work[22] and consumes the net income from his savings. Given an interest rate *r* and taxation of income from savings at a proportional rate *t*, his budget constraint in the second period is

$$C_2 = (1 + r(1 - t))E$$

Assuming perfect financial markets, the consumer can save as much as he likes, and the two budget constraints can be aggregated in an intertemporal constraint

$$C_1 + pC_2 = w$$

where *p* is the relative price of second-period consumption, that is,

$$p = \frac{1}{1 + r}$$

without taxation and

$$p = \frac{1}{1 + r(1 - t)}$$

with taxation.

As usual, the increase in *p* due to taxation has two effects:

- an income effect: the increase in *p* reduces both C_1 and C_2 if consumptions in both periods are normal goods, which increases savings $E = w - C_1$
- a substitution effect: the increase in *p* makes second-period consumption more expensive and thus tends to reduce savings.

More precisely, denote $U(C_1, C_2)$ the utility function of the consumer[23]. We can write

$$\frac{\partial C_1}{\partial p} = \left(\frac{\partial C_1}{\partial p}\right)_U - C_2 \frac{\partial C_1}{\partial w}$$

Define the intertemporal elasticity of substitution as

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22 Thus the first period could be the active period of life and the second retirement.

23 We can neglect the disutility of labor since labor supply is assumed to be inelastic.

$$\sigma = \left(\frac{\partial \log(C_1 / C_2)}{\partial \log p} \right)_U$$

First note that since Hicksian demands are the derivatives of the expenditure function $e(p,U)$ with respect to prices, the equality

$$C_1(p,U) + pC_2(p,U) = e(p,U)$$

implies by differentiating in *p* that

$$\left(\frac{\partial C_1}{\partial p} \right)_U + p\left(\frac{\partial C_2}{\partial p} \right)_U = 0$$

Since by definition

$$\sigma = \left(\frac{\partial \log C_1}{\partial \log p} \right)_U - \left(\frac{\partial \log C_2}{\partial \log p} \right)_U$$

we obtain

$$\sigma = \left(\frac{p}{C_1} + \frac{1}{C_2} \right)\left(\frac{\partial \log C_1}{\partial \log p} \right)_U = \frac{w}{pC_2}\left(\frac{\partial \log C_1}{\partial \log p} \right)_U$$

and

$$\left(\frac{\partial \log C_1}{\partial \log p} \right)_U = e\sigma$$

where $e = E / w = pC_2 / w$ denote the savings rate.

We also have

$$\frac{\partial C_1}{\partial w} = \frac{C_1}{w} \frac{\partial \log C_1}{\partial \log w}$$

Finally, by substituting within the Slutsky equation and denoting

$$\eta = \frac{\partial \log C_1}{\partial \log w}$$

the income elasticity of first-period consumption, we get

$$\frac{\partial \log C_1}{\partial \log p} = e\sigma - C_2 \frac{p}{C_1} \eta \frac{C_1}{w} = e(\sigma - \eta)$$

Moreover

$$\frac{\partial \log E}{\partial \log p} = -\frac{C_1}{E} \frac{\partial \log C_1}{\partial \log p}$$

whence

$$\frac{\partial \log E}{\partial \log p} = -(1-e)(\sigma - \eta)$$

which shows the negative substitution effect $(-(1-e)\sigma)$ and the income effect $(1-e)\eta$. What is the order of magnitude of the resulting effect? Note that once more, the Cobb-Douglas utility function is not much help since it implies $\sigma = \eta = 1$ and thus no effect of taxation on savings. A reasonable assumption is that preferences are homothetic so that both consumptions are proportional to permanent income $(\eta = 1)$. Choose $e = r = 1/2$, which is not absurd since the two periods represent the working life and retirement. Then a 50 percent tax on income from savings increases p by 20 percent and reduces savings by 10 percent multiplied by $(\sigma - 1)$ (to the first order). Thus, to get large effects of taxation on savings, the intertemporal elasticity of substitution has to be rather large. It is even quite possible that taxation increases savings (it is the case if and only if $\sigma < \eta$).

If the consumer is paid wages in both periods, then we must take into account a new income effect as permanent income becomes

$$w_1 + \frac{w_2}{1 + r(1-t)}$$

This time the consumer may decide to borrow (if his second-period wages are relatively high), which makes imperfections on financial markets relevant. If the interest rate at which he can borrow r^+ is larger than the interest rate paid on his savings r^-, then his budget constraint has a kink at the zero savings point. Under these circumstances some consumers will choose to locate in that point[24], and the substitution effect does not come into play, at least locally. This clearly reduces the negative influence of taxation on savings.

So far we have neglected the taxation of labor income. If it is taxed at the same rate as income from savings (as is the case for the ideal income tax), *w* must be replaced with $w(1-t)$. Then taxation reduces permanent income and thus both consumptions. Since savings this time is $w(1-t)-C_1$, the way this effect goes depends on the income elasticity η.

The taxation of savings affects not only income but also accumulated savings. Such is the case for wealth taxes, but also for taxes on bequests. Assume that in addition to his consumptions the consumer derives utility from any (after-tax) bequest *H* he leaves at his death. Then his utility is $U(C_1, C_2, H)$, and given a taxation rate τ on bequests, his second-period budget constraint becomes

$$C_2 + \frac{H}{1-\tau} = E(1+r(1-t))$$

His intertemporal budget constraint becomes

$$C_1 + pC_2 + p'H = w$$

where *p* is still defined as

$$p = \frac{1}{1+r(1-t)}$$

and $p' = p/(1-\tau)$. With a taxation rate of income from savings fixed at *p*, by the Hicks-Leontief theorem, the two consumptions can be aggregated within a composite good. The effect of changes on the rate of bequest taxation τ then is formally analogous to that of *t* on savings. This analysis of bequests is only half convincing, however. Whether bequests are planned or accidental (due to early deaths) is a controversial issue. In any case the taxation of bequests, like wealth taxes, collects very small amounts of tax revenues in most countries. In the United States the tax on bequest is sometimes called a voluntary tax, as it is fairly easy to avoid.

Empirical Results

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[24] They are liquidity constrained: they consume their income within each period.

In the 1970s econometricians tried to estimate the elasticity of aggregate consumer savings to the after-tax interest rate. Apart from Boskin (1978) who obtained a value close to 0.4, most estimates were close to zero. This quasi-consensus was shaken by a paper of Summers (1981). Using the calibration of a life-cycle model in a growing economy, Summers showed that any choice of parameters compatible with the observed ratio of wealth to income implied a large elasticity of savings to the interest rate. More recent work, however, has showed that Summers's result is fragile.

The literature turned in the 1980s to the estimation of Euler equations derived from the intertemporal optimization of consumers; this yielded values for the intertemporal elasticity of substitution σ. Studies done on macroeconomic data have yielded small values for σ. More credible estimations on individual data suggest that σ is nonnegligible but lower than one (which is its value for a Cobb-Douglas utility function), which implies a very small elasticity of savings to the interest rate.

Finally, many authors have used the existence of investments that are favored by taxation. Most of these studies use data from the United States, where it is possible to use Individual Retirement Accounts (IRA) and 401(k) funds to save into pension funds and deduct the amount saved from taxable income. These funds have been very successful, but the important question is whether the money that went into them would have been saved anyway or not. The studies are not unanimously conclusive, but it seems that total savings was only moderately stimulated by the favorable tax treatment of these funds.

A general lesson of this literature[25] is that taxation is unlikely to have a large impact on total savings, although it clearly plays an important role in determining where the money is invested.

1.3.3 Taxation and Risk-Taking

Taxation is often said to discourage risk-taking, since it confiscates part of the return to risky activities such as setting up a business or investing in shares. Domer-Musgrave (1944) however noted that taxation transforms government into a sleeping partner who absorbs part of the risk, which may in fact encourage risk-taking. We will revisit this argument following Mossin. We will set aside the question of whether such risk-taking as exists in the economy is too large or too small – popular opinion is that risk-taking is insufficient and should be encouraged, but there is no good evidence either for or against this view.

We consider the portfolio choice of investing in a riskless asset that brings a return r and a risky asset that brings a random return x[26]. We therefore assume that there exists a safe asset

[25] Bernheim (2002) contains a much more detailed discussion.

[26] To make the problem nontrivial, we assume that $Ex>r$ and that r lies in the interior of the support of x.

in the economy, which is an approximation (even the purchasing power of money is affected by inflation). We also assume that *two-fund separation* holds: all risky assets may be aggregated in a single composite risky asset[27].

The investor has a strictly concave von Neumann-Morgenstern utility function *u* on wealth strictement concave, meaning that he is risk-averse. We denote W_0 the initial wealth and W the final wealth. If *a* is the proportion of the initial wealth invested in the risky asset, then

$$W = (1-\tau)W_0(1+(ax+(1-a)r)(1-t))$$

where τ is the tax rate on wealth and *t* is the tax rate on asset income.

The investor maximizes $Eu(W)$ in *a*, which gives the first-order condition

$$E(u'(W)(x-r)) = 0$$

Taxation enters this expression via final wealth *W*. The impact of taxation of wealth is easy to see, since it just multiplies initial wealth W_0 by $(1-\tau)$. We know from Arrow (1970) that if absolute risk-aversion $-u''(W)/u'(W)$ is nonincreasing in wealth[28], then the amount invested in the risky asset increases with wealth, which means in our case that $a(1-\tau)$ is a decreasing function of τ. To go further and to conclude that the *proportion a* invested in the risky asset is reduced by the taxation of wealth, we need to ensure that *a* increases with wealth. This is only true if relative risk-aversion decreases with wealth, which is not clear from the empirical evidence.

Now assume that wealth is not taxed $(\tau = 0)$ and focus on taxation of asset income. The first-order condition is

$$E(u'(W_0)(1+(ax+(1-a)r)(1-t)))(x-r)) = 0 \qquad (1)$$

Let us differentiate it wit hrespect to *t*. We get

$$E\left(u''(W)(x-r)\left((x-r)(1-t)\frac{\partial a}{\partial t} - (ax+(1-a)r)\right)\right) = 0$$

whence by rearranging

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27 Two-fund separation was used for the first time by Tobin (1958). It can be justified under rather strict assumptions on preferences (see Cass-Stiglitz (1970)).

28 This so-called NIARA (*nonincreasing absolute risk-aversion*) hypothesis is confirmed by almost all empirical studies.

$$-\frac{\partial \log a}{\partial \log(1-t)} = 1 + \frac{r}{a}\frac{Eu''(W)(x-r)}{E(u''(W)(x-r)^2)} \qquad (2)$$

First note an interesting special case: if $r = 0$ (e.g., if the riskless asset is money in a world without inflation), we find that

$$-\frac{\partial \log a}{\partial \log(1-t)} = 1$$

which shows that $a(1-t)$ is independent of t and therefore implies that taxation increases a. The intuition is that of Domar-Musgrave: taxation amounts to a participation of government in risk and therefore encourages risk-taking.

When $r \neq 0$, things are slightly more complicated. Taxation of income from the riskless asset indeed reduces wealth and may change the attitude of the investor toward risk. To evaluate the second term in (2), we must define the amount invested in the risky asset $Z = aW_0$ and study how it changes with wealth. Rewriting the first-order condition (1) with this new notation, we get

$$E(u'(W_0)(1+r(1-t)) + (1-t)(x-r)Z)(x-r)) = 0$$

or by differentiating with respect to initial wealth W_0,

$$E\left(u''(W)(x-r)\left(1+r(1-t)+(1-t)+(x-r)\frac{\partial Z}{\partial W_0}\right)\right) = 0$$

Rearranging obtains

$$\frac{\partial \log Z}{\partial \log W_0} = -\frac{Eu''(W)(x-r)}{E(u''(W)(x-r)^2)}\frac{1+r(1-t)}{a(1-t)} \qquad (3)$$

Substituting (3) in (2) finally yields

$$-\frac{\partial \log a}{\partial \log(1-t)} = 1 - \frac{\partial \log Z}{\partial \log W_0}\frac{r(1-t)}{1+r(1-t)}$$

Assume that $r > 0$. We saw that under the NIARA hypothesis, Z increases in W_0.

Therefore there is a new wealth effect that induces an increase in risk-aversion and thus makes risk-taking less appealing than in the $r = 0$ case. Arrow thought that the elasticity of Z in W_0 must be lower than one. If such is the case, then the right-hand side is still positive and taxation must always encourage risk-taking, but Arrow's hypothesis is controverted.

Note that until now we implicitly assumed that the government shared in losses as well as gains. This assumption can be justified if losses can be deducted from gains on other risky assets and the resulting gain is always positive. Other wise, it is useful to examine the impact of the *no loss offset* rule, whereby the government does not subsidize losses. Then $x(1-t)$ must be replaced with x when $x < 0$; with $r \geq 0$, it does not change the return of the riskless asset. It is easy to see that when t gets close to one, then taxation must always reduce risk-taking: it does not reduce losses and gains become negligible. In general, taking the *no loss offset* rule into account tends to reduce risk-taking relative to the case where the government also takes its share of the losses.

Finally, note that in most countries, capital gains are taxed at a lower rate than interest income. Then one should consider that x and r are in fact $x(1-t')$ and $r(1-t)$, with $t' < t$. If risk-aversion does not vary too much with wealth, then this tends to increase risk-taking relative to a uniform taxation.

These remarks show that the real world is more complex than appears from the model. Moreover taxation of asset income is one of the most intricate areas of existing tax systems. Since household data usually are not very detailed on portfolio holdings, this makes estimating the effect of taxation on the holdings of risky assets very difficult. The survey of Poterba (2002) nevertheless concludes that taxation has substantial effects on how households allocate their wealth.

1.3.4 Welfare Losses

The preceding section shows that taxes change the economic behavior of private agents in ways that may be more complicated than popular wisdom suggests. Can we quantify the welfare losses induced by these distortions more generally than in the simple example that opened this chapter?

Conceptually the problem is rather simple. Start from an economy charatcterized by a tax system t_0 (and maybe other distortions), with an after-tax price equilibrium p_0. Now change the tax system to t_1 and denote the new after-tax equilibrium prices p_1. Who bears the burden of taxes? Here we focus on welfare losses, also called *deadweight losses* or *excess burdens*, that is, on the total weight of this tax burden.

Consider a "simple" example where taxes are purely redistributive, with no public good to be financed. Take a consumer *i*. Given after-tax prices *p* and taxes *t*, his utility can be written from his indirect utility function:

$$U_i(p,t) = V(p,(p-t)\cdot w_i + \sum_{j=1}^{Jx} \theta_{ij}\pi_j(p-t) + T_i(t))$$

where the θ_{ij} are his shares of firms' profits π_j and $T_i(t)$ represents the value of taxes that are redistributed to him. We would like to evaluate a sum of changes in utility such as $\sum_{i=1}^{n}(U_i(p_1,t_1) - U_1(p_0,t_0))$ but this makes no sense in general since utilities are ordinal.

Even if we neglect this first difficulty, computing the U_i must take into account all general equilibrium interactions, which seems a hopeless task. To simplify the problem futther, consider a representative consumer with an income *R* that is unchanged by taxation; then take $t_0 = 0$ and introduce a tax $t_1 = t$ on some good. The after-tax prices move from p_0 to p_1 after the introduction of tax *t*. In the general case, changes in utility can be evaluated using the equivalent variation or the compensating variation (e.g., see Salanié 2000, Ch.2). The equivalent variation equivalent, for instance, is by definition

$$E = e(p_0, V(p_1, R)) - R$$

where $e(p,u)$ is the expenditure function, that is, the amount that must be spent at prices *p* to reach utility level *u*.

The equivalent variation therefore is the amount that must be given to the consumer before introducing the tax so that he gets exactly the after-tax utility level (of course, *E*<0 when *t*>0). The consumer thus loses –*E* from the introduction of the tax, the producers lose $\left(\sum_j \pi_j(p_0) - \sum_j \pi_j(p_1 - t)\right)$, and the government collects $tx(p_1, R)$. In these conditions it seems reasonable to define the welfare loss as the sum of what the consumer and the producers lose, minus the tax revenue collected by the government (which may be redistributed). The resulting expression is

$$R - e(p_0, V(p_1, R)) + \left(\sum_j \pi(p_0) - \sum_j \pi(p_1 - t)\right) - tx(p_1, R)$$

This may appear to be a satisfactory solution. However, using the compensating variation instead of the equivalent variation would give a different measure of the consumer's welfare loss and therefore of the social welfare loss. The only case where two measures coincide is when the marginal utility of some good (denoted *m*) is constant, or

$$U = u(x) + m$$

As is well know, this amounts to the assumption that there is no income effect. Then the equivalent variation and the compensating variation both equal the Dupuit-Marshall measure of consumer surplus.

Let us now adopt all of these very restrictive assumptions (no general equilibrium effects, no income effect, representative consumer, a starting position with neither taxes nor distortions). We will now examine the effect of introducing an infinitesimal specific tax *dt* on a good.

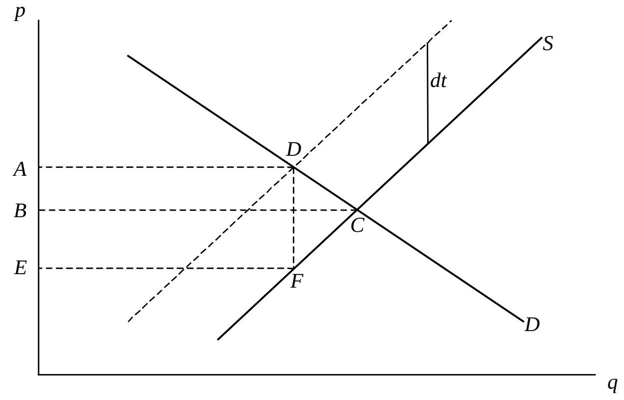
In figure 6, *p* represents the consumer price. The tax shifts the supply curve upward by *dt*. The consumer surplus thus decreases by *ABCD*, the producers' profit by *BCFE*, and the government collects *ADFE*. The social welfare loss is just the (curved) triangle *DFC*, which has basis *dt* and height (-*dx*), where *dx* is the change in the quantity traded. Since the surface of a triangle equals the half-product of its basis and its height, the social welfare loss is $-dtdx/2$. Note that it is positive whatever the sign of *dt*, for a subsidy as well as for a tax.

We already know from section 1.2 that

$$\frac{dx}{x} = -\frac{\varepsilon_D \varepsilon_S}{\varepsilon_D + \varepsilon_S}\frac{dt}{p}$$

Since *dx* is proportional to *dt*, the deadweight loss is proportional to the square of the tax, as Dupuit noted as early as 1844.

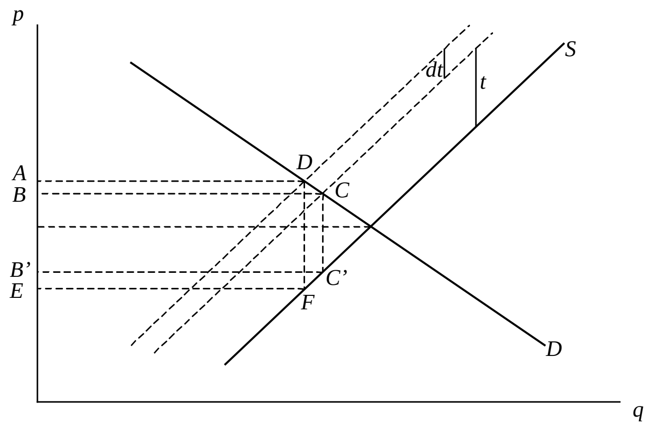
Figure 6 Second-order welfare loss



This is often invoked as an argument for tax smoothing, the idea that to collect a given revenue, it is better to have several small taxes than one big tax. This idea can be applied to the financing of government expenditure over time: for a given intertemporal tax revenue, it is better to keep tax rates constant (and have a pattern of surpluses and deficits) than to have them vary across years with budgetary needs. Contrariwise, this argument should be applied to taxation of several goods with some caution.

Also note that the tax revenue collected by the government is *xdt*, meaning that the ratio of the deadweight loss to the tax revenue is proportional to the tax rate. To give an order of magnitude, consider the "normal" rate of VAT in the European Union, which is about 20 percent, and assume that demand is unit-elastic. If supply is also unit-elastic, then the deadweight loss is about 5 percent of tax revenue, which is not negligible. If production exhibits constant returns, then $\varepsilon_S = +\infty$ and the deadweight loss goes up to 10 percent of tax revenue. This example, even though it is purely illustrative, shows that the ratio of the deadweight loss to the tax revenue, which is often called the social cost of public funds, takes values high enough that looking for a tax system that minimizes distortions is a useful task[29].

Figure 7 First-order welfare loss



What if we start from a tax rate $t > 0$ to go to $(t + dt)$? Then in figure 7 the consumer loses *ABCD*, the producers lose *B'C'FE*, and the government collects *AEFD*, instead of *BB'CC'* before the tax hike. The deadweight loss now is *DFC'C*, which is easily seen to be equal to *–tdx*; thus it is this time of first order in *dt*.

We saw in this chapter that in general equilibrium and with income effects, there is no universal definition for social welfare losses. Let us, however, close the argument by summing up the very elegant paper by Debreu (1954), which uses the coefficient of resource utilization. Start from an economy without distortions (an thus without taxes) where the

[29] Estimates of the social cost of public funds are usually obtained in the literature on CGE models. They vary a lot according to the tax that is studied, but they range from 10 to 50 percent.

vector of initial resources is ω. Now introduce taxes. The new equilibrium leads to utilities U_i. Debreu defines the coefficient of resource utilization $0 < p < 1$ as the smallest number *f* such that in the original economy with initial resources multiplied by *r*, there exists a Pareto optimum where each consumer *i* has a utility level at least equal to U_i. We can then define the inefficiency induced by taxes as $(1-p)$, and the social welfare loss by $(1-p)p \cdot \omega$, where *p* is a price vector that supports the original Pareto optimum. Debreu gives an expression for this social welfare loss that generalizes the expression in $-dtdx/2$ that we obtained in a much more restrictive model. In the very simple example that opens this chapter ,it is easy to compute that the coefficient of resource utilization is

$$p = \frac{2\sqrt{c(c+t)}}{2c+t}$$

and that the social welfare loss is

$$1-p = \frac{t^2}{(2c+t)(2c+t+2\sqrt{c(c+t)})}$$

whereas surplus analysis gives the approximation

$$\frac{t^2}{8c^2}$$

which is equivalent for small *t*.

1.4 Conclusion

To conclude this chapter, recall that there are two elasticity concepts: compensated elasticities and uncompensated elasticities. The compensated elasticites only account for substitution effects, while the uncompensated elasticities also take into account income effects. Even lump-sum transfers, which we know induce no distortion and no social welfare loss, create income effects – this is indeed their role in the second welfare theorem. Distortions and social welfare losses are entirely imputable to substitution effects, and therefore their evaluation involves compensated elasticities[30]. On the other hand, the effects of taxation on behavior

[30] The analysis in the preceding section assumed away income effects, so that compensated and uncompensated elasticities coincided there.

involve both substitution effects and income effects, and therefore they should be measured using uncompensated elasticities.

Appendix: Some Basic Concepts

The concerns of equity and efficiency may be integrated into a general framework.

Pareto efficient tax systems – tax structures such that, given the tools and information available to the government, no one can be made better off without making someone else worse off. Then we choose among the possible Pareto efficient tax structures using a social welfare function, which summarizes society's attitudes toward the welfare of different individuals. Almost all would agree that if a tax structure could be found in which everyone was better off, it should be adopted.

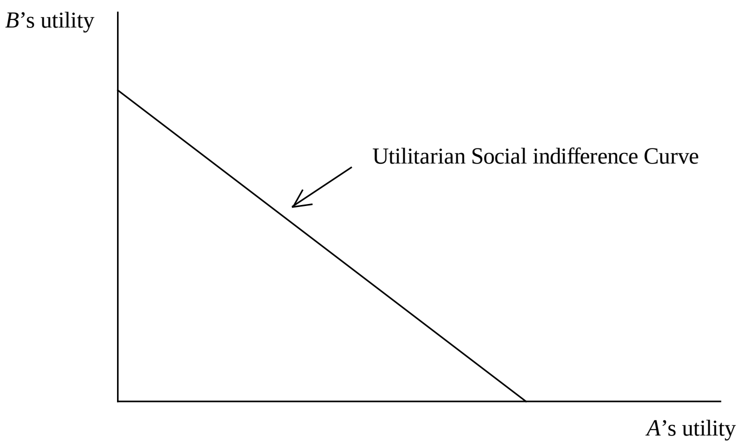
On the other hand, none of the alternative tax systems available dominates the others. In one tax system the poor may be better off, the rich worse off; but are the gains to the poor sufficiently large to justify the losses to the rich? The answer depends on value judgements, over which reasonable people may differ.

We will consider two classes of social welfare functions: *the utilitarian* (social welfare equals the sum of all individuals's utilities) and *the Rawlsian* (social welfare equals the utility of the worst-off individuals).

Utilitarianism

Taxes should be such that the marginal utility of income – the loss in utility from taking a dollar away from an individual – should be the some for all individuals. Alternative definition is as follows; society should maximize the sum of the utilities of its members such that max $w = \sum u_i$.

Figure 8

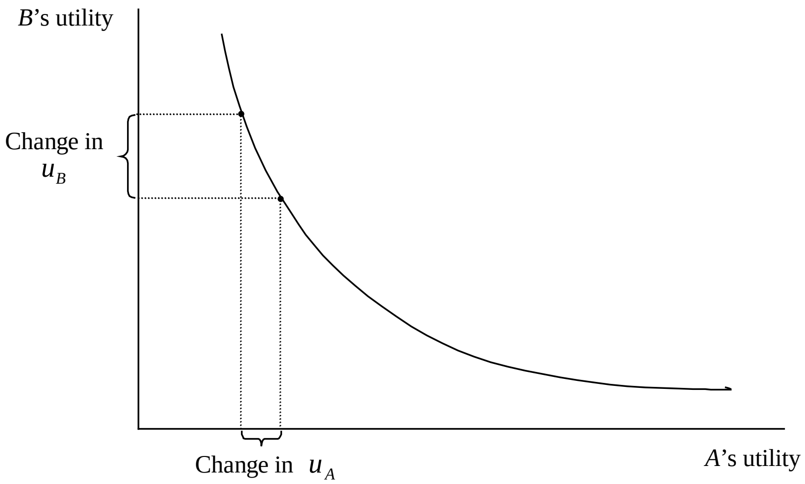


It is important to emphasize that with a utilitarian social welfare function, society is not indifferent to an increase of one dollar for individual *A* and a decrease of one dollar for individual *B*. If individual *A* has a lower level of income than individual *B*, then the increase in utility of individual *A* from one more dollar will be greater than the decrease in utility for individual *B*.

What the utilitarian social welfare function says is that the utility of any individual should be weighted equally to the utility of any other individuals.

When an individual is worse off than another, society is not in different to a decrease in the utility of the poorer (individual *A*) matched by an equal increase in the utility of the richer (individual *B*). Society should be willing to accept a decrease in the utility of the poor only if there is a much larger increase in the utility of the rich (see Figure 9).

Figure 9

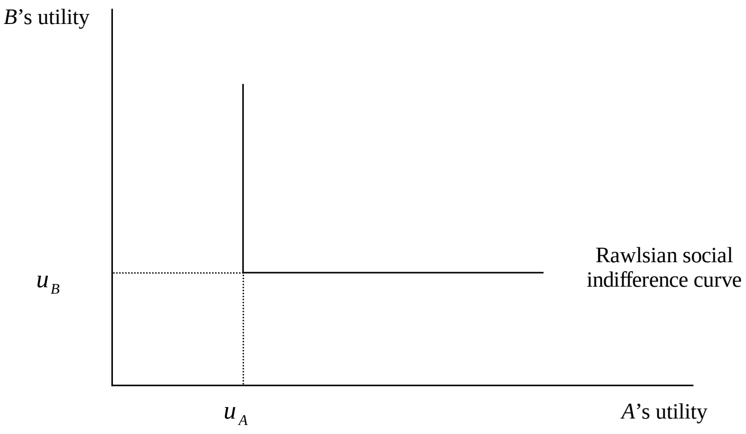


The poor individual becomes worse and worse off, the increment in utility of the richer individual that makes society indifferent must be larger and larger.

Rawlsianism

The welfare of society only depends on the welfare of the worse-off individual; society is better off if you improve his welfare but gains nothing from improving the welfare of others. There is no trade-off consideration in this social welfare function (social indifference curve takes *L*-shape).

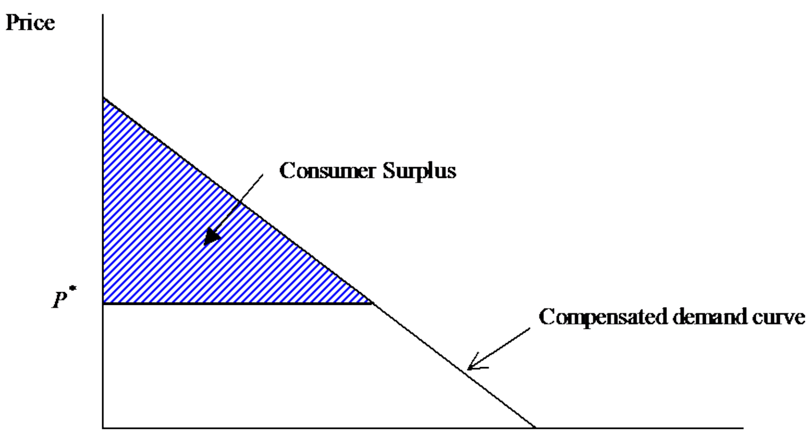
Figure 10



Concumer Surplus

The difference between what an individual is willing to pay and what he has to pay is called his *consumer surplus*. Diagrammatically, the consumer surplus is depicted in Figure 11 as the shaded area under the compensated demand curve and above the price line.

Figure 11 Graphical Representation of Consumer Surplus



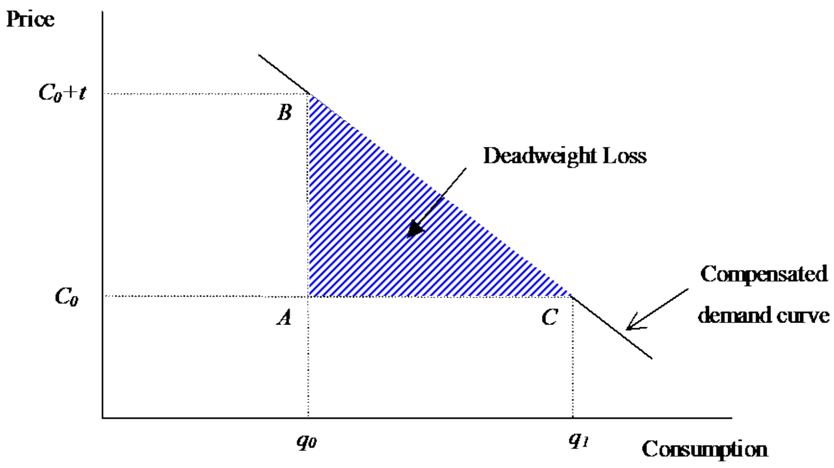
There are several ways that economists go about trying to measure consumer surplus and willingness to pay.

Deadweight Loss

To measure the economic inefficiency, economists use exactly the same methodology they use to measure the dollar value of a new project. Economists ask, "how much would an individual be willing to give up to have the inefficiency eliminated?".

Consider the inefficiency caused by a tax on cigarettes. We ask each individual how much he would be willing to pay to have the tax on cigarettes eliminated. His answer can be $100. Then eliminating the cigarette tax and imposing in its place a $100 **lump-sum tax** leaves his welfare unchanged. The difference between the revenue raised by the cigarette tax (say, $80) and the lump-sum tax that the individual would be willing to pay is called the **deadweight loss** or **excess burden** of the tax. It is the measure of the inefficiency of the tax. Figure 12 illustrates the case in point.

Figure 12 Measuring Inefficiencies



The cost of producing a cigarette is C_0, and the tax raises the price from C_0 to $C_0 + t$, where t is the tax per pack. Assume also the individual consumers q_0 packs of cigarettes with the tax, and q_1 after the tax has been removed. The deadweight loss is measured by the shaded area ABC which is sometimes called a *Harberger Triangle*.

Distributional Effects

There are many groups in society, and each may be affected differently. Some poor individuals may be hurt, some helped. In practice, governments focus on a few summary measures of inequality. Sine the poor are of particular concern, they receive special attention. The *poverty index* measures the fraction of the population whose income lies below a critical threshold; below that threshold, individuals are considered to be in poverty.

Another measure is the *poverty gap*. The poverty index only counts the number of individuals who are below the poverty threshold; it does not look at how far below that threshold they are. We need to calculate how much we need to eliminate individuals under the poverty line.

Exercises

1. (Hindriks and Myles (2006), chapter 12, Exercise 12.4)
   Assume that the preferences of the social planner are given by the function $W = \frac{[U^1]^\varepsilon}{\varepsilon} + \frac{[U^2]^\varepsilon}{\varepsilon}$. What effect does an increase in ε have on the curvature of a social indifference curve? Use this result to relate the value of ε to the planner's concern for equity).
2. (Hindriks and Myles (2006), chapter 12, Exercise 12.5)
   There are H consumers who each have utility function $U^h = \log(M^h)$. If the social welfare function is given by $W = \sum U^h$, show that a fixed stock of income will be allocated equitably. Explain why this is so.
3. (Hindriks and Myles (2006), chapter 12, Exercise 12.7)
   The two consumers that constitute an economy have utility functions $U^1 = x_1^1 x_2^1$ and $U^2 = x_1^2 x_2^2$.
   a. Graph the indifference curves of the consumers, and show that at every Pareto-efficient allocation $\frac{x_1^1}{x_2^1} = \frac{x_1^2}{x_2^2}$.
   b. Employ the feasibility conditions and the result in part a to show that Pareto-efficiency requires $\frac{x_1^2}{x_2^2} = \frac{\omega_1}{\omega_2}$, where ω_1 and ω_2 denote the endowments of the two goods.
   c. Using the utility function of consumer 2, solve for x_1^2 and x_2^2 as functions of ω_1, ω_2 and U^2.
   d. Using the utility function of consumer 1, express U^1 as a function of ω_1, ω_2 and U^2.
   e. Assuming that $\omega_1 = 1$ and $\omega_2 = 1$, plot the utility possibility frontier.
   f. Which allocation maximizes the social welfare function $W = U^1 + U^2$?
4. (Hindriks and Myles (2006), chapter 12, Exercise 12.14)
   Consider a two-good exchange economy with two types of consumers. Type A have the utility function $U^A = 2\log(x_1^A) + \log(x_2^A)$ and an endowment of 3 units of good 1 and k units of good 2. Type B have the utility function $U^B = \log(x_1^B) + 2\log(x_2^B)$ and an endowment of 6 units of good 1 and $21-k$ units of good 2.
   a. Find the competitive equilibrium outcome and show that the equilibrium price $p^* = \frac{p_1}{p_2}$ of good 1 in terms of good 2 is $p^* = \frac{21+k}{15}$.
5. (Hindriks and Myles (2006) chapter 13, Exercise 13.2)
   Let the utility function be $U = 40d^{1/2}\log(M)$, where d is family size. Construct the

equivalence scale for the value of $U = 10$. How is the scale changed if $U = 20$?

6. (Hindriks and Myles (2006) chapter 13, Exercise 13.3)
   What economies of scale are there in family size? Are these greater or smaller at low incomes?
7. (Hindriks and Myles (2006) chapter 13, Exercise 13.4)
   Take the utility function $U = \log(\frac{x_1}{d}) + \log(\frac{x_2}{d})$, where d is family size and good 1 is food.
   a. What proportion of income is spent on food? Can this provide the basis for an equivalence scale? Calculate the exact equivalence scale. Does it depend on *U*?
   b. Repeat part a for the utility function $U = \left[\frac{x_1}{d}\right]^{1/2} + [x_2]^{1/2}$.
8. (Hindriks and Myles (2006) chapter 13, Exercise 13.6)
   Consider a community with ten persons.
   a. Plot the Lorenz curve for the income distribution
      (2, 4, 6, 8, 10, 12, 14, 16, 18, 20)
   b. Consider an income redistribution that takes two units of income from each of the four richest consumers and gives two units to each of the four poorest. Plot the Lorenz curve again to demonstrate that inequality has decreased.
   c. Show that the Lorenz curve for the income distribution
      (2, 3, 5, 9, 11, 12, 15, 17, 19, 20),
      crosses the Lorenz curve for the distribution in part a.
   d. Show that the two social welfare functions $W = \sum M^h$ and $W = \sum \log(M^h)$ rank the income distributions in parts a and c differently.
9. (Hindriks and Myles (2006) chapter 13, Exercise 13.7)
   What is the Gini index, and how can it be used to determine the impact of taxes and transfers on income inequality?
10. (Hindriks and Myles (2006, chapter 13, Exercise 13.13)
    Discuss the following quote from Cowell (1995): "The main disadvantage of G[ini] is that an income transfer from a rich to a poorer man has a much greater effect on *G* if the men are near the middle rather than at either end of the parade." Do you agree? Why or why not? (Hint: Use the formula for the Gini coefficient to determine the effect of a fixed transfer at different points in the income distribution). Does the Gine have other "disadvantages"?
11. (Hindriks and Myles (2006, chapter 13, Exercise 13.14)
    Consider a hypothetical island with only ten people. Eight have income of $10,000, one

has income of $50,000, and one has income of $100,000.

a. Draw the Lorenz curve for this income distribution. What is the approximate value of the Gini coefficient?
b. Suppose that a wealthy newcomer arrives on this island with an income of $500,000. How does it change the Lorenz curve? What is the impact on the Gini coefficient?

References

Arrow, K. (1970) *Essays in the Theory of Risk-Bearing*, North-Holland.

Ballentine, J., and I Eris. (1975) "On the general equilibrium analysis of tax incidence." *Journal of Political Economy*, 83: 633-44.

Bernheim, D. (2002) "Taxation and savings." in *Handbook of Public Economics*, vol.3, A. Auerbach and M. Feldstein, eds. North-Holland.

Blundell, R. and T. MaCurdy (1999) "Labor supply: A review of alternative approaches." in the *Handbook of Labor Economics*, vol.3, O. Ashenfelter and D. Card, eds. North-Holland.

Boskin, M. (1978) "Taxation, saving, and the rate of interest." *Journal of Political Economy*, 86: S3-S27.

Cass, D. and J. Stiglitz (1970) "The structure of investor preferences and asset returns, and separability in portfolio allocation." *Journal of Economic Theory*, 2: 122-60.

Debreu, G. (1954) "A classical tax-subsidy problem." *Econometrica*, 22: 14-22.

Domar, E. and R. Musgarve (1944) "Proportional income taxation and risk-taking." *Quarterly Journal of Economics*, 58: 388-422.

Dupuit, J. (1844) "De la mesure de l'utilité des travaux publics." *Annales des Ponts et Chaussées*, 332-75. Published in English in P. Jackson, ed. *The Foundations of Public Finance*. Elgar, Cheltenham, England, 1996.

Eissa, N. (1995) "Taxation and labour supply of married women: The Tax Reform Act of 1986 as a natural experiment." *NBER Working Paper*, 5023.

Fullerton, D., and D. Rogers. (1993) *Who Bears the Lifetime Tax Burden?*, The Brookings Institution.

Harberger, A. (1962) "The incidence of the corporation tax." *Journal of Political Economy*, 70: 215-40.

Heckman J. (1979) "Sample Selection Bias as a Specification Error" *Econometrica*, 47 (1); pp.153-62.

Hindriks, J. and G.D. Myles (2006) *Intermediate Public Economics*, The MIT Press.

Holtz-Eakin, D., D. Joulfaian and H. Rosen (1993) "The Carnegie conjecture: Some empirical evidence." *Quarterly Journal of Economics*, 108: 413-35.

Leach, John (2004) *A Course in Public Economics*, Cambridge University Press.

Lemieux, T., B. Fortin and P. Frechette (1994) "The effect of taxes on labour supply in the underground economy." *American Economic Review*, 84: 231-54.

Mossin, J. (1968) "Taxation and risk-taking: An expected utility approach." *Economica*, 137: 74-82.

Poterba, J. (2002) "Taxation, risk-taking, and household portfolio behavior." In *Handbook of Public Economics*, vol.3, A. Auerbach and M. Feldstein, eds. North-Holland.

Rosen, H.S. (1999) *Public Finance*, 5th ed., New York: McGraw-Hill.

Salanié, B. (2000) *The Microeconomics of Market Failures*, MIT Press.

Salanié, B. (2003) *The Economics of Taxation*, MIT Press.

Shoven, J., and J. Whalley. (1972) A general equilibrium calculation of the effects of differential taxation of income from capital in the US. *Journal of Public Economics*, 1: 281-321.

Shoven, J., and J. Whalley. (1984) Applied general equilibrium models of taxation and international trade: An introduction and survey. *Journal of Economic Literature*, 22: 1007-51.

Stiglitz, J.E. (2000) Economics of the Public Sector, 3rd ed., New York: W.W. Norton.

Summers, L. (1981) "Taxation and capital accumulation in a Life cycle growth model."

American Economic Review, 71, pp.533-54.
Tobin, J. (1958) "Liquidity preference as behavior towards risk." *Review of Economic Studies*, 25, pp.65-8.
Tresch, Richard, W. (2002) *Public Finance: A Normative Theory*, San Diego: Academic Press.