Chapter 1　Monetary and Fiscal Policy[1]

1.1 Introduction

A public-finance approach yields several insights.　Among the most important is the recognition that fiscal and monetary policies are linked through the government sector's budget constraint.　Variations in the inflation rate can have implications for the fiscal authority's decisions about expenditures and taxes, and, conversely, decisions by the fiscal authority can have implications for money growth and inflation.

　　When inflation is viewed as a distortionary revenue-generating tax, the degree to which it should be relied upon depends on the set of alternative taxes available to the government and on the reasons individuals hold money.　Whether the most appropriate strategy is to think of money as entering the utility function as a final good or as serving as an intermediate input into the production of transaction services can have implications for whether money should be taxed. The optimal-tax perspective also has empirical implications for inflation.

1.2 Budget Accounting

To obtain goods and services, governments in market economies need to generate revenue. And one way that they can obtain goods and services is to print money that is then used to purchase resources from the private sector.　However, to understand the revenue implications of inflation (and the inflation implications of the government's revenue needs), we must start with the government's budget constraint[2].

　　Consider the following identity for the fiscal branch of a government:

$$G_t + i_{t-1}B_{t-1}^T = T_t + (B_t^T - B_{t-1}^T) + RCB_t, \tag{1}$$

where all variables are in nominal terms.　The left side consists of government expenditures on goods ,services, and transfers G_t, plus interest payments on the outstanding debt $i_{t-1}B_{t-1}^T$ (the superscript T denoting total debt, assumed to be one period in maturity, where debt issued in

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[1]　This chapter draws from Walsh (2003, Chapter 4).
[2]　Bohn (1992) provides a general discussion of government deficits and accounting.

period $t-1$ earns the nominal interest rate i_{t-1}), and the right side consists of tax revenue T_t, plus new issues of interest-bearing debt $B_t^T - B_{t-1}^T$, plus any direct receipts from the central bank RCB_t. As an example of *RCB*, the U.S. Federal Reserve turns over to the Treasury almost all the interest earnings on its portfolio of government debt[3]. We will refer to (1) as the *Treasury's budget constraint*.

The monetary authority, or central bank, also has a budget identity that links changes in its assets and liabilities. This takes the form

$$(B_t^M - B_{t-1}^M) + RCB_t = i_{t-1}B_{t-1}^M + (H_t - H_{t-1}),\qquad(2)$$

where $B_t^M - B_{t-1}^M$ is equal to the central bank's purchases of government debt, $i_{t-1}B_{t-1}^M$ is the central bank's receipt of interest payments from the Treasury, and $H_t - H_{t-1}$ is the change in the central bank's own liabilities. These liabilities are called *high-powered money* or sometimes the *monetary base* since they form the stock of currency held by the nonbank public plus bank reserves, and they represent the reserves private banks can use to back deposits under a fractional reserve system. Changes in the stock of high-powered money lead to changes in broader measures of the money supply, measures that normally include various types of bank deposits as well as currency held by the public.

By letting $B = B^T - B^M$ be the stock of government interest-bearing debt held by the public, the budget identities of the Treasury and the central bank can be combined to produce the consolidated government-sector budget identity:

$$G_t + i_{t-1}B_{t-1} = T_t + (B_t - B_{t-1}) + (H_t - H_{t-1}).\qquad(3)$$

From the perspective of the consolidated government sector, only debt held by the public (i.e., outside the government sector) represents an interest-bearing liability.

According to (3), the dollar value of government purchases G_t, plus its payment of interest on outstanding privately held debt $i_{t-1}B_{t-1}$, must be funded by revenue that can be obtained from one of three alternative sources. First, T_t represents revenues generated by taxes (other than inflation). Second, the government can obtain funds by borrowing from the private sector.

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[3] In 2001, the Federal Reserve banks turned over $27 billion to the Treasury (*88nd Annual Report of the Federal Reserve System* 2001, p.383). Klein and Neumann (1990) show how the revenue generated by seigniorage and the revenue received by the fiscal branch may differ.

This borrowing is equal to the change in the debt held by the private sector, $B_t - B_{t-1}$. Finally, the government can print currency to pay for its expenditures, and this is represented by the change in the outstanding stock of noninterest-bearing debt, $H_t - H_{t-1}$.

We can divide (3) by $P_t Y_t$, where P_t is the price level and Y_t is real output, to obtain

$$\frac{G_t}{P_t Y_t} + i_{t-1}\left(\frac{B_{t-1}}{P_t Y_t}\right) = \frac{T_t}{P_t Y_t} + \frac{B_t - B_{t-1}}{P_t Y_t} + \frac{H_t - H_{t-1}}{P_t Y_t}.$$

Note that terms like $B_{t-1}/P_t Y_t$ can be multiplied and divided by $P_{t-1}Y_{t-1}$, yielding

$$\frac{B_{t-1}}{P_t Y_t} = \left(\frac{B_{t-1}}{P_{t-1}Y_{t-1}}\right)\left(\frac{P_{t-1}Y_{t-1}}{P_t Y_t}\right) = b_{t-1}\left[\frac{1}{(1+\pi_t)(1+\mu_t)}\right],$$

where $b_{t-1} = B_{t-1}/P_{t-1}Y_{t-1}$ represents real debt relative to income, π_t is the inflation rate, and μ_t is the growth rate of real output[4]. Employing the convention that lowercase letters denote variables deflated by the price level and by real output, the government's budget identity is

$$g_t + \bar{r}_{t-1}b_{t-1} = t_t + (b_t - b_{t-1}) + h_t - \frac{h_{t-1}}{(1+\pi_t)(1+\mu_t)}, \tag{4}$$

where $\bar{r}_{t-1} = (1+i_{t-1})/[(1+\pi_t)(1+\mu_t)]-1$ is the *ex post* real return from $t-1$ to t. For simplicity, in the following we will abstract from real income growth by setting $\mu_t = 0$.

To highlight the respective roles of anticipated and unanticipated inflation, let r_t be the *ex ante* real rate of return and let π_t^e be the expected rate of inflation; then $1+i_{t-1} = (1+r_{t-1})(1+\pi_t^e)$. Adding $(r_{t-1}-\bar{r}_{t-1})/b_{t-1} = (\pi_t - \pi_t^e)(1+r_{t-1})b_{t-1}/(1+\pi_t)$ to both sides of (4) and rearranging, the budget constraint becomes

$$g_t + r_{t-1}b_{t-1} = t_t + (b_t - b_{t-1}) + \left(\frac{\pi_t - \pi_t^e}{1+\pi_t}\right)(1+r_{t-1})b_{t-1} + \left[h_t - \left(\frac{1}{1+\pi_t}\right)h_{t-1}\right]. \tag{5}$$

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[4] If n is the rate of population growth and λ is the growth rate of real per capita output, then $1+\mu = (1+n)(1+\lambda)$.

The third term on the right side of this expression, involving $(\pi_t - \pi_t^e)b_{t-1}$, represents the revenue generated when unanticipated inflation reduces the real value of the government's outstanding interest-bearing nominal debt. To the extent that inflation is anticipated, it will be reflected in higher nominal interest rates that the government must pay. Inflation by itself does not reduce the burden of the government's interest-bearing debt; only unexpected inflation has such an effect.

The last bracketed term in (5) represents seigniorage, the revenue form money creation. Seigniorage can be written as

$$s_t \equiv \left(\frac{H_t - H_{t-1}}{P_t Y_t} \right) = (h_t - h_{t-1}) + \left(\frac{\pi_t}{1 + \pi_t} \right) h_{t-1} . \tag{6}$$

Seigniorage arises from two sources. First, $h_t - h_{t-1}$ is equal to the change in real high-powered money holdings relative to income. Since the government is the monopoly issuer of high-powered money, an increase in the amount of high-powered money that the private sector is willing to hold allows the government to obtain real resources in return. In a steady-state equilibrium, h is constant, so this source of seigniorage then equals zero. The second term in (6) is normally the focus of analyses of seigniorage because it can be nonzero even in the steady state. To maintain a constant level of real money holdings relative to income, the private sector needs to increase its nominal holdings of money at the rate π (approximately) to offset the effects of inflation on real holdings. By supplying money to meet this demand, the government is able to obtain goods and services or reduce other taxes[5].

If we denote the growth rate of the nominal monetary base H by θ, the growth rate of h will equal $(\theta - \pi)/(1 + \pi) \approx \theta - \pi$. In a steady state, h will be constant, implying that $\pi = \theta$ [6]. In this case, (6) shows that seigniorage will equal

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[5] With population and real income growth, (6) becomes $s_t = (h_t - h_{t-1}) + \left[\dfrac{(1 + \pi_t)(1 + n_t)(1 + \lambda_t) - 1}{(1 + \pi_t)(1 + n_t)(1 + \lambda_t)} \right] h_{t-1}$

where n is the rate of population growth and λ is the rate of per capita income growth. Private sector nominal money holdings increase to offset inflation and population growth. In addition, if the elasticity of real money demand with respect to income is equal to 1, real per capita demand for money will rise at the rate λ. Thus, the demand for nominal balances rises approximately at the rate $\pi + n + \lambda$ when h is constant.

[6] With population and income growth, the growth rate of h is approximately equal to $\theta - \pi - n - \lambda$. In the steady state, this equals zero, or $\pi = \theta - n - \lambda$.

$$\left(\frac{\pi}{1+\pi}\right)h = \left(\frac{\theta}{1+\theta}\right)h \ . \tag{7}$$

For small values of the rate of inflation, $\pi/(1+\pi)$ is approximately equal to π, so s can be thought of as the product of a tax rate of π, the rate of inflation, and a tax base of h, the real stock of base money. Since base money does not pay interest, its real value is depreciated by inflation whether inflation is anticipated or not.

The definition of s would appear to imply that the government receives no revenue if inflation is zero. But this inference neglects the real interest savings to the government of issuing h, which is noninterest-bearing debt, as opposed to b, which is interest-bearing debt. That is, for a given level of the government's total real liabilities $d = b + h$, interest costs will be a decreasing function of the fraction of this total that consists of h. A shift from interest-bearing to noninterest-bearing debt would allow the government to reduce total tax revenues or increase transfers or purchases.

This observation suggests that one should consider the government's budget constraint expressed in terms of the total liabilities of the government. Using (5) and (6), we can rewrite the budget constraint as[7]

$$g_t + r_{t-1}d_{t-1} = t_t + (d_t - d_{t-1}) + \left(\frac{\pi_t - \pi_t^e}{1+\pi_t}\right)(1+r_{t-1})d_{t-1} + \left(\frac{i_{t-1}}{1+\pi_t}\right)h_{t-1} \ . \tag{8}$$

Seigniorage, defined as the last term in (8), becomes

$$\bar{s} = \left(\frac{i}{1+\pi}\right)h \ . \tag{9}$$

This shows that the relevant tax rate on high-powered money depends directly on the nominal rate of interest. Thus, under the Friedman rule for the optimal rate of inflation, which calls for setting the nominal rate of interest equal to zero, the government collects no revenue from seigniorage. The budget constraint also illustrates that any change in seigniorage requires an offsetting adjustment in the other components of (8). Reducing the nominal interest rate to zero implies that the lost revenue must be replaced by an increase in other taxes, real borrowing

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[7] To obtain this, add $r_{t-1}h_{t-1}$ to both sides of (5)

that increases the government's net indebtedness, or reductions in expenditures.

The various forms of the government's budget identity suggest at least three alternative measures of the revenue governments generate through money creation. First, the measure that might be viewed as appropriate from the perspective of the Treasury is simple *RCB*, total transfers from the central bank to Treasury (see Equation (1)). For the United States, King and Plosser (1985) report that the real value of these transfers amounted to 0.02% of real GNP during the 1929-1952 period and 0.15% of real GNP in the 1952-1982 period. Under this definition, shifts in the ownership of government debt between the private sector and the central bank affect the measure of seigniorage even if high-powered money remains constant. That is, from (2), if the central bank used interest receipts to purchase debt, B^M would rise, *RCB* would fall, and the Treasury would, from (1), need to raise other taxes, reduce expenditures, or issue more debt. But this last option means that the Treasury could simply issue debt equal to the increase in the central bank's debt holdings, leaving private debt holdings, government expenditures, and other taxes unaffected. Thus, changes in RCB do not represent real changes in the Treasury's finances and are therefore not the appropriate measure of seigniorage.

A second possible measure of seigniorage is given by (6), the real value of the change in high-powered money. King and Plosser report that *s* equaled 1.37% of real GNP during 1929-1952 but only 0.3% during 1952-1982. This measure of seigniorage equals the revenue from money creation for a given path of interest-bearing government debt. That is, *s* equals the total expenditures that could be funded, holding constant other tax revenues and the total private sector holdings of interest-bearing government debt. While *s*, expressed as a fraction of GNP, was quite small during the postwar period in the United States, King and Plosser report much higher values for other countries. For example, it was more than 6% of GNP in Argentina and over 2% in Italy.

Finally, (9) provides a third definition of seigniorage as the nominal interest savings from issuing noninterest-bearing as opposed to interest-bearing debt[8]. Using the four-to six-month commercial paper rate as a measure of the nominal interest rate, King and Plosser report that this measure of seigniorage equaled 0.2% of U.S. GNP during 1929-1952 and 0.47% during 1952-1982. This third definition equals the revenue from money creation for a given path of total (interest-and noninterest-bearing) government debt; it equals the total expenditures that could e funded, holding constant other tax revenues and the total private sector holdings of real government liabilities.

The difference between *s* and $\bar{s}$ arises from alternative definitions of fiscal policy. To

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[8] And these are not the only three possible definitions. See King and Plosser (1985) for an additional three.

understand the effects of monetary policy, we normally want to consider changes in monetary policy while holding fiscal policy (and perhaps other things also) constant. Suppose tax revenues t are simply treated as lump-sum taxes. Then one definition of fiscal policy would be in terms of a time series for government purchases and interest-bearing debt: $\{g_{t+i}, b_{t+i}\}_{i=0}^{\infty}$. Changes in s, together with the changes in t necessary to maintain $\{g_{t+i}, b_{t+i}\}_{i=0}^{\infty}$ unchanged, would constitute monetary policy. Under this definition, monetary policy would change the total liabilities of the government (i.e. , $b+h$). An open market purchase by the central bank would, ceteris paribus, lower the stock of interest-bearing debt held by the public. The Treasury would then need to issue additional interest-bearing debt to keep the b_{t+i} sequence unchanged. Total government liabilities would rise. Under the definition $\bar{s}$, fiscal policy sets the path $\{g_{t+i}, d_{t+i}\}_{i=0}^{\infty}$ and monetary policy determines the division of d between interest- and noninterest-bearing debt but not its total.

Intertemporal Budget Balance

The budget relationships derived in the previous section link the governemnt's choices concerning expenditures, taxes, debt, and seigniorage at each point in time. However, unless there are restrictions on the governmnet's ability to borrow or to raise revenue from seigniorage, (8) places no real constraint on expenditure or tax choices. If governments, like individuals, are constrained in their ability to borrow, then this constraint limits the government's choices. To see exactly how it does so requires that we focus on the intertemporal budget constraint of the government.

Ignoring the effect of surprise inflation, the single-period budget identity of the government given by (5) can be written as

$$g_t + r_{t-1}b_{t-1} = t_t + (b_t - b_{t-1}) + s_t .$$

Assuming the interest factor r is a constant (and is positive)[9], this equation can be solved forward to obtain

$$(1+r)b_{t-1} + \sum_{i=0}^{\infty} \frac{g_{t+i}}{(1+r)^i} = \sum_{i=0}^{\infty} \frac{t_{t+i}}{(1+r)^i} + \sum_{i=0}^{\infty} \frac{s_{t+i}}{(1+r)^i} + \lim_{i \to \infty} \frac{b_{t+i}}{(1+r)^i} . \qquad (10)$$

The government's expenditure and tax plans are said to satisfy the requirement of intertemporal

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[9] With population growth and trend income growth, the relevant discount factor is $r - n - \lambda$.

budget balance (the *no Ponzi condition*) if the last term in (10) equals zero:

$$\lim_{i \to \infty} \frac{b_{t+i}}{(1+r)^i} = 0 \, . \tag{11}$$

In this case, the right side of (10) becomes the present discounted value of all current and future tax and seigniorage revenues, and this is equal to the left side, which is the present discounted value of all current and future expenditures plus current outstanding debt (principal plus interest). In other words, the government must plan to raise sufficient revenue, in present value terms, to repay its existing debt and finance its planned expenditures. Defining the primary deficit as $\Delta = g - t - s$, intertemporal budget balance implies, from (10), that

$$(1+r)b_{t-1} = \sum_{i=0}^{\infty} \frac{\Delta_{t+i}}{(1+r)^i} \, . \tag{12}$$

Thus, if the government has outstanding debt ($b_{t-1} > 0$), the present value of future primary deficits must be negative (i.e., the government must run a primary surplus in present value). This surplus can be generated through adjustments in expenditures, taxes, or seigniorage.

1.3 Financing Government Expenditures

We now consider alternative ways of financing government expenditures and some of the implications for fiscal policy. We ignore money finance and focus on tax and debt finance. The balanced-budget multiplier, a well-known result derived from the traditional Keynesian model, is that a tax-financed permanent increase in government expenditure permanently raises output and consumption. We consider whether this result also holds in our dynamic general equilibrium model. We also examine the effects of temporary fiscal policies and whether using debt finance makes a difference. For simplicity, we ignore issues related to money and inflation, and hence we assume that the interest rate is constant.

Tax Finance

Consider first a permanent increase of Δg_t in government expenditures from period t that is financed by an increase in lump-sum taxes of ΔT_t in period t. Nothing that $b_{t-1} = b_t$ in all of

the following examples, the GBCs (Government Budget Constraints) for periods $t-1$, t, and $t+1$ are

$$t-1: \quad g_{t-1} + Rb_t = T_{t-1},$$
$$t: \quad g_{t-1} + \Delta g_t + Rb_t = T_{t-1} + \Delta T_t,$$
$$t+1: \quad g_{t-1} + \Delta g_t + Rb_t = T_{t-1} + \Delta T_t.$$

Thus, if government expenditures are raised permanently by Δg_t, then taxes must be raised permanently by the same amount to satisfy the GBC. We now examine the effect on consumption and GDP.

We have seen previously that consumption in the DGE model is proportional to wealth, and not income as in the Keynesian model. If inflation is zero, consumption and household wealth are

$$c_t = \frac{R}{1+R} W_t \quad ,$$
$$W_t = \sum_{s=0}^{\infty} \frac{(x_{t+s} - T_{t+s})}{(1+R)^s} + (1+R)b_t \quad ,$$

where x_t is income before taxes, which is assumed to be exogenous. If future income and taxes are expected to remain at their time t and $t-1$ levels, then $x_{t+s} = x_t = x_{t-s}$ and $T_{t+s} = T_t$ for all $s \geq 0$. This implies that consumption is determined by current total income:

$$c_t = x_t - T_t + Rb_t \quad . \tag{13}$$

We now introduce a permanent increase in government expenditures in period t. Since taxes also increase permanently by ΔT, consumption in periods $t-1$ and t is

$$c_{t-1} = x_{t-1} - T_{t-1} + Rb_t \quad ,$$
$$c_t = x_t - (T_{t-1} + \Delta T_t) + Rb_t$$
$$= c_{t-1} - \Delta T_t$$
$$= c_{t-1} - \Delta g_t \quad .$$

The increase in government expenditure has therefore been fully offset by a reduction in private consumption due to a fall in wealth caused by extra taxes. If the national income identity at time $t-1$ is

$$y_{t-1} = c_{t-1} + g_{t-1} \quad,$$

then

$$y_t = c_{t-1} + \Delta c_t + g_{t-1} + \Delta g_t = y_{t-1} \quad,$$

As $\Delta c_t = -\Delta g_t$, GDP is unchanged. The fiscal stimulus has therefore been totally ineffective as the injection of expenditure has been completely crowded out by expected increases in taxes.

If the fiscal expenditure increase takes the form of an increase in transfers, then higher taxes would completely offset the higher transfer income. There would therefore be no change in wealth, consumption, or GDP because, for unchanged income x_t and a permanent increase in transfers of Δh_t, wealth in period t would be

$$w_t = \frac{(1+R)(x_t + h_{t-1} + \Delta h_t - T_{t-1} - \Delta T_t)}{R} + (1+R)b_t = w_{t-1} \quad.$$

We contrast this result with the standard Keynesian balanced-budget multiplier. The standard Keynesian consumption function assumes that consumption is a proportion $0 < \mu < 1$ of total income, so that instead of equation (13) we have

$$c_t = \mu(x_t - T_t + Rb_t) \quad.$$

It then follows that after the expenditure increase,

$$\begin{aligned} y_t &= \mu(x_t - T_{t-1} - \Delta T_t + Rb_t) + g_{t-1} + \Delta g_t \\ &= y_{t-1} + (1-\mu)\Delta g_t > y_{t-1} \quad. \end{aligned}$$

Thus, if $0 < \mu < 1$, then GDP would increase and fiscal policy would be effective in the Keynesian model.

Bond Finance

We now assume that pure bond finance is used. Issuing more bonds raises government

expenditures through the additional interest payments. We distinguish between a permanent and a temporary increase in government expenditures.

A Permanent Increase of Δg_t *in period* t

The sequence of government budget constraints in periods $t-1$, t, $t+1$, ets., following a permanent increase in government expenditures is

$$
\begin{aligned}
t-1: \quad & g_{t-1} + Rb_t = T_{t-1} \quad , \\
t: \quad & g_t + \Delta g_t + Rb_t = T_{t-1} + \Delta b_{t+1} \quad , \\
t+1: \quad & g_t + \Delta g_t + R(b_t + \Delta b_{t+1}) = T_{t-1} + \Delta b_{t+2} \quad , \\
& \qquad\qquad \vdots \\
t+n-1: \quad & g_t + \Delta g_t + Rb_t + R\sum_{s=1}^{n-1}\Delta b_{t+s} = T_{t-1} + \Delta b_{t+n} \quad .
\end{aligned}
$$

Hence,

$$
\Delta b_{t+n} = (1+R)^{n-1}\Delta g_t
$$

and so

$$
b_{t+n} = b_t + \sum_{s=1}^{n}\Delta b_{t+s} = b_t + (1+R)\left[\frac{(1+R)^{n-1}-1}{R}\right]\Delta g_t .
$$

Therefore,

$$
\frac{b_{t+n}}{(1+R)^n} = \frac{b_t}{(1+R)^n} + \left[\frac{1}{R} - \frac{1}{R(1+R)^{n-1}}\right]\Delta g_t
$$

and

$$
\lim_{n\to\infty}\frac{b_{t+n}}{(1+R)^n} = \frac{1}{R}\Delta g_t \neq 0 .
$$

As discounted debt is not zero, it follows that debt grows without bound. This violates the intertemporal budget constraint. Hence, a bond-financed permanent increase in government

expenditures is not sustainable.

A Temporary Increase of Δg_t *(Or a Fall in T) Only in Period* t

The sequence of government budget constraints in periods $t-1$, t, $t+1$, ets., is now

$$
\begin{aligned}
t-1: \quad & g_{t-1} + Rb_t = T_{t-1} \quad , \\
t: \quad & g_{t-1} + \Delta g_t + Rb_t = T_{t-1} + \Delta b_{t+1} \quad , \\
t+1: \quad & g_{t-1} + R(b_t + \Delta b_{t+1}) = T_{t-1} + \Delta b_{t+2} \quad , \\
& \qquad\qquad\qquad \vdots \\
t+n-1: \quad & g_{t-1} + R\left(b_t + \sum_{s=1}^{n} \Delta b_{t+s} \right) = T_{t-1} + \Delta b_{t+n} \quad ,
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta b_{t+1} &= \Delta g_t \quad , \\
\Delta b_{t+2} &= R\Delta g_t \quad , \\
\Delta b_{t+3} &= R(1+R)\Delta g_t \quad , \\
& \qquad\quad \vdots \\
\Delta b_{t+n} &= R(1+R)^{n-2}\Delta g_t \quad ,
\end{aligned}
$$

Hence

$$
\begin{aligned}
b_{t+n} &= b_t + \sum_{s=1}^{n} \Delta b_{t+s} \\
&= b_t + \left[1 + R \sum_{s=0}^{n-2} (1+R)^s \right] \Delta g_t \\
&= b_t + (1+R)^{n-1}\Delta g_t \quad , \\
\frac{b_{t+n}}{(1+R)^n} &= \frac{b_t}{(1+R)^n} + \frac{1}{1+R}\Delta g_t \quad ,
\end{aligned}
$$

and so

$$
\lim_{n\to\infty} \frac{b_{t+n}}{(1+R)^n} = \frac{1}{1+R}\Delta g_t \neq 0 .
$$

As discounted debt is not zero, fiscal policy is still not sustainable.

Suppose, however, that the temporary change in government expenditures was a random

shock, and that in each period there is a random shock with zero mean, we can then write $\Delta g_t = e_t$, where $E(e_t) = 0$ and $E(e_t e_{t+s}) = 0$. As a result, we now consider the *average* discounted value of debt, which is

$$\lim_{n \to \infty} E\left[\frac{b_{t+n}}{(1+R)^n}\right] = \frac{1}{1+R} E[\Delta g_t] = 0 .$$

Hence debt no longer explodes. We have shown, therefore, that bond-financing temporary increases in government expenditures that are expected to be zero on average (i.e., fiscal policy shocks) is a sustainable policy because positive shocks are expected to be offset over time by negative shocks.

A similar argument can be made with respect to the business cycle, where the shocks may be serially correlated over time. If increases in expenditures during periods of recession are offset by decreases in expenditures during boom, and if these cancel out over a complete cycle, then debt finance can be used. In particular, there is no need to raise taxes during a recession as many governments seem to do, just because the government deficit is increasing. It is, however, important that, when fiscal surpluses reappear in the upturn, these are used to redeem debt and are not used to cut taxes. There are often strong pressures on government to cut taxes during a boom, but it may be necessary to resist these to avoid the accumulation of debt across cycles and hence to keep public finances on a sustainable path in the longer term.

Intertemporal Fiscal Policy

Suppose the government wants to provide a temporary stimulus to the economy. One possible policy is to cut taxes today, finance this by borrowing today, and then, as the stimulus is temporary, to restore tax revenues tomorrow. In this way fiscal policy will be sustainable. What is the effect of this on the GBC and on consumption?

Let the tax cut occur in period t, and assume that in period $t+2$ the GBC of period $t-1$ is to be restored. Consequently, the GBCs for periods $t-1$, t, $t+1$, and $t+2$ are

$$t-1: \quad g_{t-1} + Rb_t = T_{t-1} \quad ,$$
$$t: \quad\quad g_{t-1} + Rb_t = T_{t-1} + \Delta T_t + \Delta b_{t+1} \quad ,$$
$$t+1: \quad g_{t-1} + R(b_t + \Delta b_{t+1}) = T_{t-1} + \Delta T_{t+1} + \Delta b_{t+2} \quad ,$$
$$t+2: \quad g_{t-1} + Rb_t = T_{t-1} \quad ,$$

where, due to the tax cut, $\Delta T_t < 0$.   Thus

$$\Delta b_{t+1} = -\Delta T_t \quad ,$$
$$\Delta b_{t+2} = -\Delta b_{t+1} \quad ,$$
$$\Delta T_{t+1} = R\Delta b_{t+1} - \Delta b_{t+2} = -(1+R)\Delta T_t \quad ,$$

Hence, for b_{t+2} to be restored to b_{t-1}, taxes must be increased in period $t+1$ by $-(1+R)\Delta T_t$.

It can be shown that wealth is unaffected by this as its values in periods $t-1$ and t are

$$W_{t-1} = \sum_{s=0}^{\infty} \frac{x_{t+s-1} - T_{t+s-1}}{(1+R)^s} + (1+R)b_{t-1} \quad ,$$

$$W_t = \sum_{s=0}^{\infty} \frac{x_{t+s} - T_{t+s}}{(1+R)^s} + (1+R)b_t$$

$$= W_{t-1} - \Delta T_t - \frac{1-(1+R)}{(1+R)}\Delta T_t = W_{t-1} - \frac{1}{1+R}\Delta T_t \quad .$$

As $\Delta T_t < 0$, wealth and hence consumption increase in period t.   In period $t+1$ wealth returns to its period $t-1$ value as higher taxes exactly offset the increase in financial assets. Thus the stimulus to consumption of the bond-financed tax cut is only temporary.

1.4 Money and Fiscal Policy Frameworks

Most analyses of monetary phenomena and monetary policy assume, usually without statement, that variations in the stock of money matter but that how that variation occurs does not.   The nominal money supply could change due to a shift from tax-financed government expenditures to seigniorage-financed expenditures.   Or it could change as the result of an open market operation in which the central bank purchases interest-bearing debt, financing the purchase by an increase in noninterest-bearing debt, holding other taxes constant (see 1.2 Budget Accounting).   Because these two means of increasing the money stock have differing implications for taxes and the stock of interest-bearing government debt, they may lead to different effects on prices and/or interest rates.

The government sector's budget constraint links monetary and fiscal policies in ways that can matter for determining how a change in the money stock affects the equilibrium price level[10].

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[10]   See, for example, Sargent and Wallace (1981) and Wallace (1981).   The importance of the budget constraint for the analysis of monetary topics is clearly illustrated in Sargent (1987).

The budget link also means that one needs to be precise about defining monetary policy as distinct from fiscal policy. An open market purchase increases the stock of money, but by reducing the interest-bearing government debt held by the public, it has implications for the future stream of taxes needed to finance the interest cost of the government's debt. So an open market operation potentially has a fiscal side to it, and this fact can lead to ambiguity in defining what one means by a change in monetary policy, *holding fiscal policy constant*.

The literature in monetary economics has analyzed several alternative assumptions about the relationship between monetary and fiscal policies. In most traditional analyses, fiscal policy is assumed to adjust to ensure that the government's inter-temporal budget is always in balance, while monetary policy is free to set the nominal money stock or the nominal rate of interest. This situation is described as a *Ricardian regime* (Sargent 1982), one of monetary dominance, or one in which fiscal policy is passive and monetary policy is active (Leeper 1991). Some models fall into this category in that fiscal policy was ignored and monetary policy determined the price level. Traditional quantity theory relationships were obtained – one-time proportional changes in the nominal quantity of money led to equal proportional changes in the price level.

If fiscal policy affects the real of interest[11], then the price level is not independent of fiscal policy, even under regimes of monetary dominance. A balanced budget increase in expenditures that raises the real interest rate raises the nominal interest rate and lowers the real demand for money. Given an exogenous path for the nominal money supply, the price level must jump up to reduce the real supply of money.

A second policy regime is one in which the fiscal authority sets its expenditure and taxes without regard to any requirement of intertemporal budget balance. If the present discounted value of these taxes is not sufficient to finance expenditures (in present value terms), seigiorage must adjust to ensure that the government's intertemporal budget constraint is satisfied. This regime is one of fiscal dominance (or active fiscal policy) and passive monetary policy, as monetary policy must adjust to deliver the level of seigniorage required to balance the government's budget. Prices and inflation are affected by changes in fiscal policy because these fiscal changes, if they require a change in seigniorage, alter the current and/or future money supply. Aiyagari and Gertler (1985), following Sargent (1982), describe this regime as *non-Ricardian*, although more recent usage describes any regime in which taxes and/or seigniorage always adjust to ensure that the government's intertemporal budget constraint is satisfied as Ricardan. Regimes of fiscal dominance are analyzed below.

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[11]   That is, if Ricardian equivalence does not hold.

Finally, a third regime that has attracted recent attention leads to what has become known as the fiscal theory of the price level (Sims 1994; Woodford 1995; 2001b; Cochrane 1998a). In this regime, the government's intertemporal budget constraint may not be satisfied for arbitrary price levels. Following Woodford (1995), these regimes are described as non-Ricardian. The intertemporal budget constraint is satisfied only at the equilibrium price level, and the government's nominal debt plays a critical role in determining the price level. The fiscal theory of the price level is analyzed in section below.

Fiscal Dominance, Deficits, and Inflation

The intertemporal budget constraint implies that any government with a current outstanding debt must run, in present value terms, future surpluses. One way to generate a surplus is to increase revenues from seigniorage, and for that reason, economists have been interested in the implications of budget deficits for future money growth. Two questions have formed the focus of studies of deficits and inflation: First, do fiscal deficits necessarily imply that inflation will eventually occur? Second, if inflation is not a necessary consequence of deficits, is it in fact a historical consequence?

The literature on the first question has focused on the implications for inflation if the monetary authority must act to ensure that the government's intertemporal budget is balanced. This interpretation views fiscal policy as set independently, so that the monetary authority is forced to generate enough seigniorage to satisfy the intertemporal budget balance condition. Leeper (1991) describes such a situation as one in which there is an active fiscal policy and a passive monetary policy. It is also described as a situation of *fiscal dominance*.

From (12), the government's intertemporal budget constraint takes the form

$$b_{t-1} = -R^{-1} \sum_{i=0}^{\infty} R^{-i} (g_{t+i} - t_{t+i} - s_{t+i}),$$

where $R=1+r$ is the gross real interest rate, $g_t - t_t - s_t$ is the primary deficit, and s_t is real seigniorage revenue. Let $s_t^f \equiv t_t - g_t$ be the primary *fiscal* surplus (i.e., tax revenues minus expenditures but excluding interest payments and seigniorage revenue). Then the government's budget constraint can be written as

$$b_{t-1} = -R^{-1} \sum_{i=0}^{\infty} R^{-i} s_{t+i}^f + R^{-1} \sum_{i=0}^{\infty} R^{-i} s_{t+i} . \tag{14}$$

The current real liabilities of the government must be financed by, in present value terms, either a fiscal primary surplus $-R^{-1} \sum_{i=0}^{\infty} R^{-i} s_{t+i}^f$ or seigniorage.

Given the real value of the government's liabilities b_{t-1}, (14) illustrates what Sargent and Wallace (1981) described as "unpleasant monetarist arithmetic" in a regime of fiscal dominance. If the present value of the fiscal primary surplus is reduced, the present value of seigniorage must rise to maintain (14). Or, for a given present value of s^f, an attempt by the monetary authority to reduce inflation and seigniorage today must lead to higher inflation and seigniorage in the future, because the present discounted value of seigniorage cannot be altered. The mechanism is straightforward; if current inflation tax revenues are lowered, the deficit grows and the stock of debt rises. This implies an increase in the present discounted value of future tax revenues, including revenues from seigniorage. If the fiscal authority does not adjust, the monetary authority will be forced eventually to produce higher inflation[12].

The literature on the second question – has inflation been a consequence of deficits historically? – has focused on estimating empirically the effects of deficits on money growth. Joines (1985) finds money growth in the United States to be positively related to major war spending but not to nonwar deficits. Grier and Neiman (1987) summarize a number of earlier studies of the relationship between deficits and money growth (and other measures of monetary policy) in the United States. That the results are generally inconclusive is perhaps not surprising, as the studies they review were all based on postwar but pre-1980 data. Thus, the samples covered periods in which there was relatively little deficit variation and in which much of the existing variation arose from the endogenous response of deficits to the business cycle as tax revenues varied procyclically[13]. Grier and Neiman do find that the structural (high-employment) deficit is a determinant of money growth. This finding is consistent with that of King and Plosser (1985), who report that the fiscal deficit does help to predict future seigniorage for the United States. They interpret this as mixed evidence for fiscal dominance.

Demopoulos, Katsimbris, and Miller (1987) provide evidence on debt accommodation for eight OECD countries. These authors estimate a variety of central bank reaction functions

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[12] In a regime of monetary dominance, the monetary authority can determine inflation and seigniorage, in which case the fiscal authority must adjust either taxes or spending to ensure that (13) is satisfied.

[13] For that reason, some of the studies cited by Grier and Neiman employed a measure of the high-employment surplus (i.e., the surplus estimated to occur if the economy had been at full employment). Grier and Neiman conclude, "The high employment deficit (surplus) seems to have a better 'batting average.'…" (p.204).

(regression equations with alternative policy instruments on the left-hand side) in which the government deficit is included as an explanatory variable. For the post-Bretton Woods period, they find a range of outcomes, from no accommodation by the Federal Reserve and the Bundesbank to significant accommodation by the Bank of Italy and the Nederlandse Bank.

Ricardian and (Traditional) Non-Ricardian Fiscal Policies Sargent and Wallace's (1981) unpleasant monetarist arithmetic reminds us that fiscal policy and monetary policy are linked. This also means that changes in the nominal quantity of money engineered through lump-sum taxes and transfers may have different effects than changes introduced through open market operations in which noninterest-bearing government debt is exchanged for interest-bearing debt. In an early contribution, Metzler (1951) argued that an open market purchase, that is, an increase in the nominal quantity of money held by the public and an offsetting reduction in the nominal stock of interest-bearing debt held by the public, would raise the price level less than proportionally to the increase in M. An open market operation would, therefore, affect the real stock of money and lead to a change in the equilibrium rate of interest. Metzler assumed that households' desired portfolio holdings of bonds and money depended on the expected return on bonds. An open market operation, by altering the ratio of bonds to money, requires a change in the rate of interest to induce private agents to hold the new portfolio composition of bonds and money. A price-level change proportional to the change in the nominal money supply would not restore equilibrium, because it would not restore the original ratio of nominal bonds to nominal money.

An important limitation of Metzler's analysis was its dependence on portfolio behavior that was not derived directly from the decision problem facing the agents of the model. The analysis was also limited in that it ignored the consequence for future taxes of shifts in the composition of the government's debt, a point made by Patinkin (1965). We have seen that the government's intertemporal budget constraint requires the government to run surpluses in present value terms equal to its current outstanding interest-bearing debt. An open market purchase by the monetary authority reduces the stock of interest-bearing debt held by the public. This reduction will have consequences for future expected taxes in ways that critically affect the outcome of policies that affect the stock of interest-bearing debt.

Sargent and Wallace (1981) have shown that the "backing" for government debt, whether it is ultimately paid for by taxes or by printing money, is important in determining the effects of debt issuance and open market operations. This finding can be illustrated following the analysis of Aiyagari and Gertler (1985). They use a two-period overlapping-generations model that

allows debt policy to affect the real intergenerational distribution of wealth. This effect is absent from the representative-agent models we have been using, but the representative-agent framework can still be used to show how the specification of fiscal policy will have important implications for conclusions about the link between the money supply and the price level[14].

In order to focus on debt, taxes, and seigniorage, set government purchases equal to zero and ignore population and real income growth, in which case the government's budget constraint takes the simplified form

$$(1 + r_{t-1})b_{t-1} = t_t + b_t + s_t ,$$ (15)

with s_t denoting seigniorage.

In addition to the government's budget constraint, we need to specify the budget constraint of the representative agent. Assume that this agent receives an exogenous endowment y in each period and pays (lump-sum) taxes t_t in period t. She also receives interest payments on any government debt held at the start of the period; these payments in real terms, are given by $(1 + i_{t-1})B_{t-1}/P_t$, where i_{t-1} is the nominal interest rate in period $t-1$, B_{t-1} is the number of bonds held at the start of period t, and P_t is the period t price level. We can write this equivalently as $(1 + r_{t-1})b_{t-1}$, where $r_{t-1} = (1 + i_{t-1})/(1 + \pi_t) - 1$ is the ex post real rate of interest. Finally, the agent has real money balances equal to $M_{t-1}/P_t = (1 + \pi_t)^{-1}m_{t-1}$ that are carried into period t from period $t-1$. The agent allocates these resources to consumption, real money holdings, and real bond purchases:

$$c_t + m_t + b_t = y + (1 + r_{t-1})b_{t-1} + \frac{m_{t-1}}{1 + \pi_t} - t_t .$$ (16)

Aiyagari and Gertler (1985) ask whether the price level will depend only on the stock of money or whether debt policy and the behavior of the stock of debt might also be relevant for price level determination. They assume that the government sets taxes to back a fraction ψ of its interest-bearing debt liabilities, with $0 \leq \psi \leq 1$. If $\psi = 1$, government interest-bearing debt is completely backed by taxes in the sense that the government commits to maintaining the present discounted value of current and future tax receipts equal to its outstanding debt

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[14] See also Wooldford (1995, 2001b, 2003) and Section 1.4 "The Fiscal Theory of the Price Level".

liabilities. Such a fiscal policy is called *Ricardian* by Sargent (1982)[15]. If $\psi < 1$, Aiyagari and Gertler characterize fiscal policy as *non-Ricardian*. To avoid confusion with the more recent interpretations of non-Ricardian regimes (see Section 1.5 "The Fiscal Theory of the Price Level"), let regimes where $\psi < 1$ be referred to as *traditional non-Ricardian* regimes. In such regimes, seigniorage must adjust to maintain the present value of taxes plus seigniorage equal to the governmnet's outstanding debt.

Let T_t now denote the present discounted value of taxes. Under the assumed debt policy, the government ensures that $T_t = \psi(1 + r_{t-1})b_{t-1}$ since $(1 + r_{t-1})b_{t-1}$ is the net liability of the government (including its current interest payment). Because T_t is a present value, we can also write

$$T_t = t_t + E_t\left(\frac{T_{t+1}}{1+r_t}\right) = t_t + E_t\left[\frac{\psi(1+r_t)b_t}{(1+r_t)}\right]$$

or $T_t = t_t + \psi b_t$. Now because $T_t = \psi(1 + r_{t-1})b_{t-1}$, it follows that

$$t_t = \psi(R_{t-1}b_{t-1} - b_t), \tag{17}$$

where $R = 1 + r$. Similarly, $s_t = (1 - \psi)(R_{t-1}b_{t-1} - b_t)$. With taxes adjusting to ensure that the fraction ψ of the government's debt liabilities is backed by taxes, the remaining fraction, $1 - \psi$, represents the portion backed by seigniorage.

Given (17), the household's budget constraint (16) becomes

$$y + (1 - \psi)R_{t-1}b_{t-1} + \frac{m_{t-1}}{1 + \pi_t} = c_t + m_t + (1 - \psi)b_t.$$

In the Ricardian case ($\psi = 1$), all terms involving the government's debt drop out; only the stock of money matters. If $\psi < 1$, however, debt does not drop out. We can then rewrite the budget constraint as $y + R_{t-1}w_{t-1} = c_t + w_t + i_{t-1}m_{t-1}/(1 + \pi_t)$, where $w = m + (1 - \psi)b$, showing that the relevant measure of household income is $y + R_{t-1}w_{t-1}$ and this is then used to purchase

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[15] It is more common for Ricardo's name to be linked with debt in the form of the Ricardian Equivalence Theorem, under which shifts between debt and tax financing of a given expenditure stream have no real effects. See Barro (1974) or Romer (2001). Ricardian Equivalence holds in the representative-agent framework we are using; the issue is whether debt policy, as characterized by ψ, matters for price-level determination.

consumption, financial assets, or money balances (where the opportunity cost of money is $i/(1+\pi)$). With asset demand depending on ψ through w_{t-1}, the equilibrium price level and nominal rate of interest will generally depend on ψ [16].

While we have derived the representative agent's budget constraint and shown how it is affected by the means the government uses to back its debt, to actually determine the effects on the equilibrium price level and nominal interest rate, we must determine the agent's demand for money and bonds and then equate these demands to the (exogenous) supplies. To illustrate the role of debt policy, assume log separable utility, $\ln c_t + \delta \ln m_t$, and consider a perfect foresight equilibrium. We know that the marginal rate of substitution between money and consumption will be set equal to $i_t/(1+i_t)$. With log utility, this implies $m_t = \delta c_t(1+i_t)/i_t$. The Euler condition for the optimal consumption path yields $c_{t+1} = \beta(1+r_t)c_t$. Using these in the agent's budget constraint,

$$y + R_{t-1}w_{t-1} = c_t + w_t + \left(\frac{i_{t-1}}{1+\pi_t}\right)\delta\left(\frac{1+i_{t-1}}{i_{t-1}}\right)\frac{c_t}{\beta(1+r_{t-1})} = \left(1+\frac{\delta}{\beta}\right)c_t + w_t .$$

In equilibrium, $c_t = y$, so this becomes $R_{t-1}w_{t-1} = (\delta/\beta)y + w_t$. If we consider the steady state, $w_t = w_{t-1} = w^{ss} = \delta y/\beta(R-1)$. But $w = [M + (1-\psi)B]/P$, so the equilibrium steady-state price level is equal to

$$P^{ss} = \left(\frac{\beta r^{ss}}{\delta y}\right)[M + (1-\psi)B] \tag{18}$$

where $r^{ss} = R - 1$.

If government debt is entirely backed by taxes $(\psi = 1)$, we get the standard result; the price level is proportional to the nominal stock of money. The stock of debt has no effect on the price level. With $0 < \psi < 1$, however, both the nominal money supply and the nominal stock of debt play a role in price level determination. Proportional changes in M and B produce proportional changes in the price level; increases in the government's total nominal debt, $M+B$, raise P^{ss} proportionately.

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[16] In this example, $c=y$ in equilibrium since there is no capital good that would allow the endowment to be transferred over time.

In a steady state, all nominal quantities and the price level must change at the same rate since real values are constant. Thus, if M grows, then B must also grow at the same rate. The real issue is whether the composition of the government's liabilities matters for the price level. To focus more clearly on that issue, let $\lambda = M/(M + B)$ be the fraction of government liabilities that consists of noninterest-bearing debt. Since open market operations affect the relative proportions of money and bonds in government liabilities, open market operations determine λ. Equation (18) can then be written as

$$ P^{ss} = \left(\frac{\beta r^{ss}}{\delta y} \right) \big[1 - \psi(1 - \lambda) \big] (M + B) . $$

Open market purchases (an increase in λ) that substitute money for bonds but leave $M + B$ unchanged raise P^{ss} when $\psi > 0$. The rise in P^{ss} is not proportional to the increase in M. Shifting the composition of its liabilities away from interest-bearing debt reduces the present discounted value of the private sector's tax liabilities by less than the fall in debt holdings; a rise in the price level proportional to the rise in M would leave households' real wealth lower (their bond holdings are reduced in real value, but the decline in the real value of their tax liabilities is only $\psi < 1$ times as large).

Leeper (1991) argues that even if $\psi = 1$ on average (that is, all debt is backed by taxes), the means used to finance shocks to the government's budget have important implications. He distinguishes between active and passive policies; in an active monetary policy and a passive fiscal policy, monetary policy acts to target nominal interest rates and does not respond to the government's debt, while fiscal policy must then adjust taxes to ensure intretemporal budget balance (i.e., a Ricardian fiscal policy). Conversely, in an active fiscal policy and a passive monetary policy, the monetary authority must adjust seigniorage revenues to ensure intertemporal budget balance, while fiscal policy does not respond to shocks to debt. Leeper shows that the inflation and debt processes are unstable if both policy authorities follow active policies, while there is price level indeterminacy if both follow passive policies.

The Government Budget Constraint and the Nominal Rate of Interest Earlier, we examined Sargent and Wallace's unpleasant monetarist arithmetic using (14). Given the government's real liabilities, the monetary authority would be forced to finance any difference between these real liabilities and the present discounted value of the government's fiscal surpluses. Fiscal considerations determine the money supply, but the traditional quantity

theory holds and the price level is proportional to the nominal quantity of money. Suppose, however, that the initial nominal stock of money is set exogenously by the monetary authority. Does this mean that the price level is determined solely by monetary policy, with no effect of fiscal policy? In fact, the following example illustrates how fiscal policy can affect the initial equilibrium price level, even when the initial nominal quantity of money is given and the government's intertemporal budget constraint must be satisfied at all price levels.

Consider a perfect foresight equilibrium. In such an equilibrium, the government's budget constraint must be satisfied and the real demand for money must equal the real supply of money. The money-in-the-utility function (MIU) model can be used, for example, to derive the real demand for money. That model implied that agents would equate the marginal rate of substitution between money and consumption to the cost of holding money, where this cost depended on the nominal rate of interest:

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = \frac{i_t}{1 + i_t}.$$

Using the utility function such that $u(c_t, m_t) = \dfrac{\left[ac_t^{1-b} + (1-a)m_t^{1-b} \right]^{\frac{1-\Phi}{1-b}}}{1-\Phi}$.

This condition implies that

$$m_t = \frac{M_t}{P_t} = \left[\left(\frac{i_t}{1+i_t} \right) \left(\frac{a}{1-a} \right) \right]^{-\frac{1}{b}} c_t.$$

Evaluated at the economy's steady state, this can be written as

$$\frac{M_t}{P_t} = f(R_{m,t}), \tag{19}$$

where $R_m = 1 + i$ is the gross nominal rate of interest and $f(R_m) = \left[a(R_m - 1)/R_m (1-a) \right]^{-\frac{1}{b}} c$. Given the nominal interest rate, (18) implies a proportional relationship between the nominal quantity of money and the equilibrium price level. If the initial money stock is M_0, then the initial price level is $P_0 = M_0 / f(R_m)$.

The government's budget constraint must also be satisfied. In a perfect-foresight equilibrium, there are no inflation surprises, so the government's budget constraint given by (5) can be written as

$$g_t + rb_{t-1} = t_t + (b_t - b_{t-1}) + m_t - \left(\frac{1}{1+\pi_t}\right)m_{t-1}. \tag{20}$$

Now consider a stationary equilibrium in which government expenditures and taxes are constant, as are the real stocks of government interest-bearing debt and money. In such a stationary equilibrium, the budget constraint becomes

$$g + \left(\frac{1}{\beta} - 1\right)b = t + \left(\frac{\pi_t}{1+\pi_t}\right)m = t + \left(\frac{\beta R_m - 1}{\beta R_m}\right)f(R_m), \tag{21}$$

where we have used the steady-state results that the gross real interest rate is $1 + r = 1/\beta$, then $R_m \equiv (1 + \pi_t)/\beta$, and real money balances must be consistent with the demand given by (18).

Suppose the fiscal authority sets g, t, and b. Then (20) determines the nominal interest rate R_m. With g, t, and b given, the government needs to raise $g + (1/\beta - 1)b - t$ in seigniorage. The nominal interest rate is determined by the requirement that this level of seigniorage of raised[17]. Because the nominal interest rate is equal to $(1 + \pi_t)/\beta$, we can alternatively say that fiscal policy determines the inflation rate. Once the nominal interest rate is determined, the initial price level is given by (18) as $P_0 = M_0/f(R_m)$, where M_0 is the initial stock of money. In subsequent periods, the price level is equal to $P_t = P_0(\beta R_m)^t$, where $\beta R_m = (1 + \pi_t)$ is the gross inflation rate. The nominal stock of money in each future period is endogenously determined by $M_t = P_t f(R_m)$. In this case, even though the monetary authority has set M_0 exogenously, the initial price level is determined by the need for fiscal solvency since the fiscal authority's budget requirement (20) determines R_m and therefore the real demand for money. The initial price level is proportional to the initial money stock, but the factor of proportionality $(1/f(R_m))$ is determined by fiscal policy, and both the rate of inflation and the path of the future nominal money supply are determined by the fiscal requirement that seigniorage equal $g + (1/\beta - 1)b - t$.

If the fiscal authority raises expenditures, holding b and t constant, then seigniorage must rise. The equilibrium nominal interest rate rises to generate this additional seigniorage[18]. With a

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[17] The nominal interest rate that raises seigniorage equal to $g + (1/\beta - 1)b - t$ may not be unique. A rise in R_m increases the tax rate on money, but it also erodes the tax base by reducing the real demand for money. A given amount of seigniorage may b raised with a low tax rate and a high base or a high tax rate and a low base.

[18] This assumes that the economy is on the positively sloped portion of the Laffer curve so that raising the tax rate

higher R_m, the real demand for money falls, and this increases the equilibrium value of the initial price level P_0, even though the initial nominal quantity of money is unchanged.

1.5 The Fiscal Theory of the Price Level

Recently, a number of researchers have examined models in which fiscal factors replace the money supply as the key determinant of the price level (see Leeper 1991; Sims 1994; Woodford 1995, 1998b, 2001; Bohn 1998b; Cochrane 1998a; Kocherlakota and Phelen 1999; Daniel 2001, the excellent discussions by Carlstrom and Fuerst 1999b and by Christiano and Fitzgerald 2000, and references they list, and the criticisms of the approach by McCallum 2001 and Buiter 2002). The *fiscal theory of the price level* raises some important issues for both monetary theory and monetary policy.

There are two ways fiscal policy might matter for the price level. First, equilibrium requires that the real quantity of money equal the real demand for money. If fiscal variables affect the real demand for money, the equilibrium price level will also depend on fiscal factors. This however, is not the channel emphasized in fiscal theories of the price level. Instead, these theories focus on a second aspect of monetary models – there may be multiple price levels consistent with a given nominal quantity of money and equality between money supply and money demand. Fiscal policy may then determine which of these is the equilibrium price level. And in some cases, the equilibrium price level picked out by fiscal factors may be independent of the nominal supply of money.

In contrast to the standard monetary theories of the price level, the fiscal theory assumes that the government's intertemporal budget equation represents *an equilibrium condition* rather than *a constraint* that must hold for all price levels. At some price levels, the intertemporal budget constraint would be violated. Such price levels are not consistent with equilibrium. Given the stock of nominal debt, the equilibrium price level must ensure that the government's intertemporal budget is balanced.

The next subsection illustrates why the requirement that the real demand for money equal the real supply of money may not be sufficient to uniquely determine the equilibrium price level, even for a fixed nominal money supply. The subsequent subsection shows how fiscal considerations may serve to pin down the equilibrium price level.

Multiple Equilibria The tradition quantity theory of money highlights the role the nominal

increases revenue.

stock of money plays in determining the equilibrium price level. Using the demand for money given by (18), we obtained a proportional relationship between the nominal quantity of money and the equilibrium price level that depended on the nominal rate of interest. However, the nominal interest rate is also an endogenous variable, so (18) by itself may not be sufficient to determine the equilibrium price level. Because the nominal interest rate depends on the rate of inflation, (18) can be written as

$$\frac{M_t}{P_t} = f\left(R_t, \frac{P_{t+1}}{P_t}\right),$$

where R is the gross real rate of interest. As we saw in section 1.3, this forward difference equation in the price level may be insufficient to determine a unique equilibrium path for the price level.

Consider a perfect-foresight equilibrium with a constant nominal supply of money, M_0. Suppose the real rate of return is equal to its steady-state value of $1/\beta$, and the demand for real money balances is given by (18). We can then write the equilibrium between the real supply of money and the real demand for money as

$$\frac{M_0}{P_t} = g\left(\frac{P_{t+1}}{P_t}\right), \quad g' < 0.$$

Under suitable regularity conditions on $g(\bullet)$, this condition can be rewritten as

$$P_{t+1} = P_t g^{-1}\left(\frac{M_0}{P_t}\right) \equiv \phi(P_t). \tag{22}$$

Equation (21) defines a difference equation in the price level. One solution is $P_{t+i} = P^*$ for all $i \geq 0$, where $P^* = M_0 g(1)$. In this equilibrium, the quantity theory holds, and the price level is proportional to the money supply.

This constant price level equilibrium is not, however, the only possible equilibrium. As we saw in section 1.3, there may be equilibrium price paths starting from $P_0 \neq P^*$ that are fully consistent with the equilibrium condition (21). For example, in Figure 1, the convex curve shows $\phi(P_t)$ as an increasing function of P_t. Also shown in Figure 1 is the $45°$ line.

Using the fact that $g^{-1}(M_0/P^*)=1$, the slope of $\phi(P_t)$, evaluated at P^*, is

$$\phi'(P^*) = g^{-1}(M_0/P^*) - [\partial g^{-1}(M_0/P^*)/\partial(M_0/P^*)](M_0/P^*)$$
$$= 1 - [\partial g^{-1}(M_0/P^*)/\partial(M_0/P^*)](M_0/P^*) > 1$$

Thus, ϕ cuts the $45°$ line from below at P^*. Any price path starting at $P_0 = P' > P^*$ is consistent with (21) and involves a positive rate of inflation. As the figure illustrates, $P \to \infty$, but the equilibrium condition (21) is satisfied along this path. As the price level explodes, real money balances go to zero. But this is consistent with private agents' demand for money because inflation, and therefore nominal interest rates, are rising, lowering the real demand for money. Any price level to the right of P^* is a valid equilibrium. These equilibria all involve speculative hyperinflations (Equilibria originating to the left of P^* eventually violate a transversality condition since M/P is exploding as $P \to 0$). By itself, (21) is not sufficient to uniquely determine the equilibrium value of P_0, even though the nominal quantity of money is fixed.

Figure 1   Equilibrium with a Fixed Nominal Money Supply
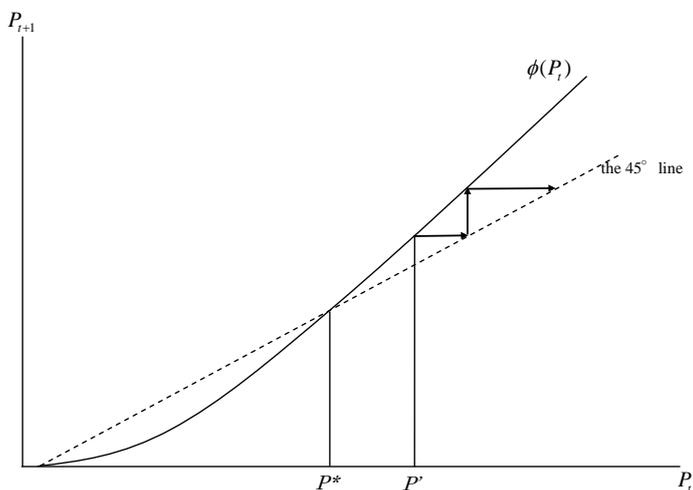
The Basic Idea   Standard models in which equilibrium depends on forward-looking expectations of the price level, generally have multiple equilibria. Thus, an additional equilibrium condition may be needed to uniquely determine the price level. The fiscal theory of the price level focuses on situations in which the government's intertemporal budget

constraint may supply the additional equilibrium condition.

The fiscal theory can be illustrated in the context of a model with a representative household and a government, but with no capital. The implications of the fiscal theory will be earliest to see if attention is restricted to perfect-foresight equilibria.

The representative household chooses its consumption and asset holdings optimally, subject to an intertemporal budget constraint. Suppose the period t budget constraint of the representative household takes the form

$$D_t + P_t y_t - T_t \geq P_t c_t + M_t^d + B_t^d = P_t c_t + \left(\frac{i_t}{1+i_t}\right) M_t^d + \left(\frac{1}{1+i_t}\right) D_{t+1}^d,$$

where D_t is the household's beginning-of-period financial wealth and $D_{t+1}^d = (1+i_t) B_t^d + M_t^d$. The superscripts denote that M^d and B^d are the household's demand for money and interest-bearing debt. In real terms, this budget constraint becomes

$$d_t + y_t - \tau_t \geq c_t + m_t^d + b_t^d = c_t + \left(\frac{i_t}{1+i_t}\right) m_t^d + \left(\frac{1}{1+r_t}\right) d_{t+1}^d,$$

where $\tau_t = T_t / P_t$, $m_t^d = M_t^d / P_t$, $1+r_t = (1+i_t)(1+\pi_{t+1})$, and $d_t = D_t / P_t$. Let

$$\lambda_{t,t+i} = \prod_{j=1}^{i} \left(\frac{1}{1+r_{t+j}}\right)$$

be the discount factor, with $\lambda_{t,t} = 1$. Under standard assumptions, the household intertemporal budget constraint takes the form

$$d_t + \sum_{i=0}^{\infty} \lambda_{t,t+i} (y_{t+1} - \tau_{t+i}) = \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[c_{t+i} + \left(\frac{i_{t+i}}{1+i_{t+i}}\right) m_{t+i}^d \right]. \tag{23}$$

Household choices must satisfy this intertemporal budget constraint. The left side is the present discounted value of the household's initial real financial wealth and after tax income. The right side is the present discounted value of consumption spending plus the real cost of holding money. This condition holds with equality because any path of consumption and money holdings for which the left side exceeded the right side would not be optimal; the

household could increase its consumption at time t without reducing consumption or money holdings at any other date. As long as the household is unable to accumulate debts that exceed the present value of its resources, the right side cannot exceed the left side.

The budget constraint for the government sector, in nominal terms, takes the form

$$P_t g_t + (1 + i_{t-1}) B_{t-1} = T_t + M_t - M_{t-1} + B_t . \tag{24}$$

Dividing by P_t, this can be written as

$$g_t + d_t = \tau_t + \left(\frac{i_t}{1+i_t}\right) m_t + \left(\frac{1}{1+r_t}\right) d_{t+1} .$$

Recursively substituting for future values of d_{t+i}, this budget constraint implies that

$$d_t + \sum_{i=0}^{\infty} \lambda_{t,t+i} [g_{t+1} - \tau_{t+i} - \bar{s}_{t+i}] = \lim_{T \to \infty} \lambda_{t,t+T} d_T , \tag{25}$$

where $\bar{s}_t = i_t m_t / (1+i_t)$ is the government's real seigniorage revenue. In previous sections, we assumed that the expenditures, taxes and seigniorage choices of the consolidated government (the combined monetary and fiscal authorities) were constrained by the requirement that $\lim_{T \to \infty} \lambda_{t,t+T} d_T = 0$ for *all* price levels P_t. Policy paths for $(g_{t+i}, \tau_{t+i}, \bar{s}_{t+i}, d_{t+i})_{i \geq 0}$ such that

$$d_t + \sum_{i=0}^{\infty} \lambda_{t,t+i} [g_{t+1} - \tau_{t+i} - \bar{s}_{t+i}] = \lim_{T \to \infty} \lambda_{t,t+T} d_T = 0$$

for all price paths p_{t+i}, $i \geq 0$ are called Ricardian policies. Policy paths for $(g_{t+i}, \tau_{t+i}, \bar{s}_{t+i}, d_{t+i})_{i \geq 0}$ for which $\lim_{T \to \infty} \lambda_{t,t+T} d_T$ may not equal zero for all price paths are called *non-Ricardian*[19].

Now consider a perfect-foresight equilibrium. Regardless of whether the government follows a Ricardian or a non-Ricardian policy, equilibrium in the goods market in this simple

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[19] Notice that this usage differs somewhat from the way Sargent (1982) and Aiyagari and Gertler (1985) employed the terms. In these earlier papers, a Ricardian policy was one in which the fiscal authority fully adjusted taxes to ensure intertemporal budget balance for all price paths. A non-Ricardian policy was a policy in which the monetary authority was required to adjust seigniorage to ensure intertemporal budget balance for all price paths. Both of these policies would be labeled Ricardian under the current usage of the term.

economy with no capital requires that $y_t = c_t + g_t$. The demand for money must also equal the supply of money: $m_t^d = m_t$. Substituting $y_t - g_t$ for c_t and m_t for m_t^d in (22) and rearranging yields

$$d_t + \sum_{i=0}^{\infty} \lambda_{t,t+i}\left[g_{t+1} - \tau_{t+i} - \left(\frac{i_{t+i}}{1+i_{t+i}}\right)m_{t+i}\right] = 0.$$ (26)

Thus, an implication of the representative household's optimization problem and market equilibrium is that (25) must hold in equilibrium. Under Ricardian policies, (25) does not impose any additional restrictions on equilibrium since the policy variables are always adjusted to ensure that this condition holds. Under a non-Ricardian policy, however, it does impose an additional condition that must be satisfied in equilibrium. To see what this condition involves, we can use the definition of d_t and seigniorage to write (25) as

$$\frac{D_t}{P_t} = \sum_{i=0}^{\infty} \lambda_{t,t+i}[\tau_{t+i} + \bar{s}_{t+i} - g_{t+1}].$$ (27)

At time t, the government's outstanding nominal liabilities D_t are predetermined by past policies. Given the present discounted value of the government's future surpluses (the right side (26)), the only endogenous variable is the current price level P_t. The price level must adjust to ensure that (26) is satisfied.

Equation (26) is an equilibrium condition under non-Ricardian policies, but it is not the only equilibrium condition. It is still the case that real money demand and real money supply must be equal. Suppose the real demand for money is given by (18), rewritten here as

$$\frac{M_t}{P_t} = f(1+i_t).$$ (28)

Equation (26) and (27) must both be satisfied in equilibrium. However, which two variables are determined jointly by these two equations depends on the assumptions that are made about fiscal and monetary policies. For example, suppose the fiscal authority determines g_{t+i} and τ_{t+i} for all $i \geq 0$, and the monetary authority pegs the nominal rate of interest $i_{t+i} = \bar{\imath}$ for all $i \geq 0$. Seigniorage is equal to $\bar{\imath}f(1+\bar{\imath})/(1+\bar{\imath})$ and so is fixed by monetary policy. With this

specification of monetary and fiscal policies, the right side of (26) is given. Since D_t is predetermined at date t, (26) can solved for the equilibrium price level P_t^* given by

$$P_t^* = \frac{D_t}{\Sigma_{i=0}^{\infty} \lambda_{t,t+i}[\tau_{t+i} + \bar{s}_{t+i} - g_{t+i}]} \, . \tag{29}$$

The current nominal money supply is then determined by (27):

$$M_t = P_t^* f(1+\bar{i}) \, .$$

One property of this equilibrium is that changes in fiscal policy (g or τ) directly alter the equilibrium price level, even though seigniorage as measured by $\Sigma_{i=0}^{\infty} \lambda_{t,t+i}\bar{s}_{t+i}$ is unaffected[20]. The finding that the price level is uniquely determined by (28) contrasts with a standard conclusion that the price level is indeterminate under a nominal interest rate peg. This conclusion is obtained from (27): with i pegged, the right side of (27) is fixed, but this only determines the real supply of money. Any price level is consistent with equilibrium, as M then adjusts to ensure that (27) holds.

Critical to the fiscal theory is the assumption that (26), the government's intertemporal budget constraint, is an *equilibrium* condition that holds at the equilibrium price level and not a condition that must hold at all price levels. This means that at price levels not equal to P_t^*, the government is planning to run surpluses (including seigniorage) whose real value, in present discounted terms, is not equal to the government's outstanding real liabilities. The government does not need to ensure that (26) holds for all price levels. Similarly, it means that the government could cut current taxes, leaving current and future government expenditures and seigniorage unchanged, and not simultaneously plan to raise future taxes. When (26)is interpreted as a budget *constraint* that must be satisfied for all price levels, then any decision to cut taxes today (and so lower the right side of (26)) must be accompanied by planned future tax increases to leave the right side unchanged.

In standard infinite-horizon, representative-agent models, a tax cut (current and future government expenditures unchanged) has no effect on equilibrium (i.e., Ricardian equivalence holds) because the tax reduction does not have a real wealth effect on private agents. They

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[20] A change in g or τ causes the price level to jump, and this transfers resources between the private sector and the government. This transfer can also be viewed as a form of seigniorage.

recognize that in a Ricardian regime, future taxes have risen in present value terms by an amount exactly equal to the reduction in current taxes. Alternatively expressed, the government cannot engineer a permanent tax cut unless government expenditures are also cut (in present value terms). Because the fiscal theory of the price level assumes that (26) holds only when evaluated at the equilibrium price level, the government can plan a permanent tax cut. If it does, the price level must rise to ensure that the new, lower value of discounted surpluses is again equal to the real value of government debt.

An interest rate peg is just one possible policy specification. As an alternative, suppose as before that the fiscal authority sets the paths for g_{t+i} and τ_{t+i}, but now suppose that the government adjusts tax revenues to offset any variations in seigniorage. In this case, $\tau_{t+i} + \bar{s}_{t+i}$ becomes an exogenous process. Then (26) can be solved for the equilibrium price level ,independent of the nominal money stock. Equation (27) must still hold in equilibrium. If the monetary authority sets M_t, this equation determines the nominal interest rate that ensures that the real demand for money is equal to the real supply. If the monetary authority sets the nominal rate of interest, (27) determines the nominal money supply. The extreme implication of the fiscal theory (relative to traditional quantity theory results) is perhaps most stark when the monetary authority fixes the nominal supply of money: $M_{t+i} + \overline{M}$ for all $i \geq 0$. Then, under a fiscal policy that makes $\tau_{t+i} + \bar{s}_{t+i}$ an exogenous process, the price level is proportional to D_t and, for a given level of D_t, is independent of $\overline{M}$.

Empirical Evidence on the Fiscal Theory Under the fiscal theory of the price level, (26) holds at the equilibrium value of the price level. Under traditional theories of the price level, (26) holds for all values of the price level. If we only observe equilibrium outcomes, it will be impossible empirically to distinguish between the two theories. As Sims (1994, p.381) puts it, "Determinacy of the price level under any policy depends on the public's beliefs about what the policy authority would do under conditions that are never observed in equilibrium."

Canzoneri, Cumby, and Diba (2001) examine VAR evidence on th response of U.S. liabilities to a positive innovation to the primary surplus. Under a non-Ricardian policy, a positive innovation to $\tau_t + \bar{s}_t - g_t$ is negatively serially correlated. The authors argue that in a Ricardian regime, a positive innovation to the current primary surplus will reduce real liabilities. This can be seen by writing the budget constraint (23) in real terms as

$$d_{t+1} = R[d_t - (\tau_t + s_t - g_t)] . \tag{30}$$

Examining U.S. data, they find that the responses are inconsistent with a Ricardian regime. Increases in the surplus are associated with declines in current and future real liabilities, and the surplus does not display negative serial correlation.

Cochrane points out the fundamental problem with this test: both (29) and (26) must hold in equilibrium, so it can be difficult to develop testable restrictions that can distinguish between the two regimes. The two regimes have different implications only if we can observe nonequilibrium values of the price level.

Bohn (1998) has examined the U.S. deficit and debt processes and concludes that the primary surplus responds positively to the debt to GDP ratio. In other words, a rise in the debt to GDP ratio leads to an increase in the primary surplus. Thus, the surplus does adjust, and Bohn finds that it responds enough to ensure that the intertemporal budget constraint is satisfied. This is evidence that the fiscal authority seems to act in a Ricardian fashion.

Finally, there is an older literature that attempted to estimate whether fiscal deficits tended to lead to faster money growth. Such evidence might be interpreted to imply a Ricardian regime of fiscal dominance.

1.6 Friedman's Rule Revisted

The preceding analysis has gone partway toward integrating the choice of inflation with the general public finance choice of tax rates, and the discussion was motivated by Phelps's conclusion that some revenue should be raised from the inflation tax if only distortionary tax sources are available. However, this conclusion has been questioned by Kimbrough (1986a, 1986b), Faig (1988), Chari, Christiano, and Kehoe (1991, 1996), and Correia and Teles (1996, 1999)[21]. They show that there are conditions under which Friedman's rule for the optimal inflation rate – a zero nominal rate of interest – continues to be optimal even in the absence of lump-sum taxes. Mulligan and Sala-i-Martin (1997) provide a general discussion of the conditions necessary for taxing (or not taxing) money.

This recent literature integrates the question of the optimal inflation tax into the general problem of optimal taxation. By doing so, the analysis can build on findings in the optimal tax literature that identify situations in which the structure of optimal indirect taxes calls for different final goods to be taxed at the same rate or for the tax rate on goods that serve as

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[21] An early example of the use of optimal tax models to study the optimal inflation rate issue is Drazen (1979). See also Walsh (1984). A recent survey is Chari and Kehoe (1999).

intermediate inputs to be zero (see Diamond and Mirrlees 1971, Atkinson and Stiglitz 1972). An MIU approach, for example, treats money as a final good; in contrast, a shopping time model, or a more general model in which money serve to produce transaction services, treats money as an intermediate input. Thus, it is important to examine what implications these alternative assumptions about the role of money might have for the optimal tax approach to inflation determination, and how optimal inflation tax results might depend on particular restrictions on preferences or on the technology for producing transaction services.

The Basic Ramsey Problem The problem of determining the optimal structure of taxes to finance a given level of expenditures is called the *Ramsey problem*, after the classic treatment of Frank Ramsey (1927). In the representative-agent models we have ben using, the Ramsey problem involves setting taxes to maximize the utility of the representative agent, subject to the government's revenue requirement.

The following static Ramsey problem, based on Mulligan and Sala-i-Martin (1997), can be used to highlight the key issues. The utility of the representative agent depends on consumption (c), real money balance (m), and leisure (l):

$$u = u(c, m, l).$$

Agents maximize utility subject to the following budget constraint:

$$f(n) \geq (1+\tau)c + \tau_m m, \tag{31}$$

where $f(n)$ is a standard production function, $n = 1 - l$ is the supply of labor, c is consumption, τ is the consumption tax, $\tau_m = i/(1+i)$ is the tax on money, and m is the household's holdings of real money balances. The representative agent picks consumption, money holdings, and leisure to maximize utility, taking the tax rates as given. Letting λ be the Lagrangian multiplier on the budget constraint, the first order conditions from the agent's maximization problem are

$$u_c = \lambda(1+\tau) \tag{32}$$

$$u_m + \lambda \tau_m \tag{33}$$

$$u_l = \lambda f' \tag{34}$$

From these first order conditions and the budget constraint, the choices of c, m, and l can be expressed as functions of the two tax rates: $c(\tau, \tau_m)$, $m(\tau, \tau_m)$, and $l(\tau, \tau_m)$.

The government's problem is to set τ and τ_m to maximize the representative agent's utility, subject to three types of constraints. First, the government must satisfy its budget constraint; tax revenues must be sufficient to finance expenditures. This constraint takes the form

$$\tau c + \tau_m m \geq g \,, \tag{35}$$

where g is real government expenditures. These expenditures are taken to be exogenous. Second, the government is constrained by the fact that consumption, labor supply, and real money must be consistent with the choices of private agents. That means that (32)-(34) represent constraints on the government's choices. Finally, the government is constrained by the economy's resource constraint:

$$f(1-l) \geq c + g \,. \tag{36}$$

The government's problem is to pick τ and τ_m to maximize $u(c,m,l)$ subject to (32)-(36).

There are two approaches to solving this problem. The first approach, often called the *dual approach*, employs the indirect utility function to express utility as a function of taxes. These tax rates are treated as the government's control variables, and the optimal values of the tax rates are found by solving the first order conditions from the government's optimization problem. The second approach, called the *primal approach*, treats quantities as the government's controls. The tax rates are found from the representative agent's first order conditions to ensure that private agents choose the quantities that solve the government's maximization problem. We start with the dual approach. The primal approach will be employed later in this section.

The government's problem can be written as

$$\max_{\tau, \tau_m} \{ v(\tau, \tau_m) + \mu[\tau_m m(\tau, \tau_m) + \tau c(\tau, \tau_m) - g] + \theta[f(1-l(\tau, \tau_m)) - c(\tau, \tau_m) - g] \} \,,$$

where $v(\tau, \tau_m) u[c(\tau, \tau_m), m(\tau, \tau_m), l(\tau, \tau_m)]$ is the indirect utility function, and μ and θ are Lagrangian multipliers on the budget and resource constraints. Notice that we have

incorporated the constraints represented by (32)-(34) by writing consumption, money balances, and leisure as functions of the tax rates. The first order conditions for the two taxes are

$$v_t + \mu(\tau_m m_\tau + c + \tau c_\tau) - \theta(f'l_\tau + c_\tau) \le 0$$

$$v_{t_m} + \mu(m + \tau_m m_{\tau_m} + \tau c_{\tau_m}) - \theta(f'l_{\tau_m} + c_{\tau_m}) \le 0 \,,$$

where $v_t = u_c c_\tau + u_m m_\tau + u_l l_\tau$ and $v_{t_m} = u_c c_{\tau_m} + u_m m_{\tau_m} + u_l l_{\tau_m}$. These conditions will hold with equality if the solution is an interior one with positive taxes on both consumption and money. If the left side of the second first order condition is negative optimal. From the resource constraint (36), $-f'l_x - c_x = 0$ for $x = \tau, \tau_m$ since g is fixed, so the two first order conditions can be simplified to yield

$$u_c c_\tau + u_m m_\tau + u_l l_\tau + \mu(\tau_m m_\tau + c + \tau c_\tau) \le 0 \tag{37}$$

$$u_c c_{\tau_m} + u_m m_{\tau_m} + u_l l_{\tau_m} + \mu(m + \tau_m m_{\tau_m} + \tau c_{\tau_m}) \le 0 \,. \tag{38}$$

The first three terms in the first of these equations can be written using the agent's first order conditions and the budget constraint as

$$u_c c_\tau + u_m m_\tau + u_l l_\tau + u_l\left(\frac{u_c}{u_l}c_\tau + \frac{u_m}{u_l}m_\tau + l_\tau\right) = \left(\frac{u_l}{f'}\right)\left[(1+\tau)c_\tau + \tau_m m_\tau + f'l_\tau\right].$$

However, differentiating the budget constraint by τ yields

$$c + (1+\tau)c_\tau + \tau_m m_\tau + f'l_\tau = 0 \,,$$

so $u_c c_\tau + u_m m_\tau + u_l l_\tau = -(u_l/f')c$. Thus, (37) becomes

$$\left(\frac{u_l}{f'}\right)c \ge \mu(\tau_m m_\tau + c + \tau c_\tau) \,,$$

while following similar steps implies that (38) becomes

$$\left(\frac{u_l}{f'}\right)m \geq \mu(m + \tau_m m_{\tau_m} + \tau c_{\tau_m}).$$

Hence, if the solution is an interior one with positive taxes on consumption and money holdings,

$$\frac{m}{c} = \frac{m + \tau_m m_{\tau_m} + \tau c_{\tau_m}}{\tau_m m_\tau + c + \tau c_\tau}. \tag{39}$$

To interpret this condition, note that $v_{\tau_m} = u_c c_{\tau_m} + u_m m_{\tau_m} + u_l l_{\tau_m} = -u_l m / f'$ is the effect of the tax on money on utility, while $v_\tau = u_c c_\tau + u_m m_\tau + u_l l_\tau = -u_l c / f'$ is the effect of the consumption tax on utility. Thus, their ratio, m/c, is the marginal rate of substitution between the two tax rates, holding constant the utility of the representative agent[22]. The right hand side of (39) is the marginal rate of transformation, holding the government's revenue constant. At an optimum, the government equates the marginal rates of substitution and transformation.

Our interest is in determining when *the Friedman rule*, $\tau_m = 0$, is optimal. Assume, following Friedman, that at a zero nominal interest rate, the demand for money is finite. Since the tax on consumption must be positive if the tax on money is zero (since the government does need to raise revenue), (37) will hold with equality. Then

$$\frac{m}{c} \geq \frac{m + \tau_m m_{\tau_m} + \tau c_{\tau_m}}{\tau_m m_\tau + c + \tau c_\tau}. \tag{40}$$

or

$$\frac{m}{m + \tau_m m_{\tau_m} + \tau c_{\tau_m}} \geq \frac{c}{\tau_m m_\tau + c + \tau c_\tau}.$$

The left side is proportional to the marginal impact of the inflation tax on utility per dollar of revenue raised. The right side is proportional to the marginal impact of the consumption tax on utility per dollar of revenue raised. If the inequality is strict at $\tau_m = 0$, then the distortion caused by using the inflation tax (per dollar of revenue raised) exceeds the cost of raising that

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[22] That is, if $v(\tau, \tau_m)$ is the utility of the representative agent as a function of the two tax rates, then

$$v_\tau d\tau + v_{\tau_m} d\tau_m = 0 \quad \text{yields} \quad \frac{d\tau}{d\tau_m} = -\frac{v_{\tau_m}}{v_\tau} = -\frac{m}{c}.$$

same revenue with the consumption tax.   Thus, it is optimal to set the tax on money equal to zero if

$$\frac{m}{c} \geq \frac{m + \tau c_{\tau_m}}{c + \tau c_\tau} \tag{41}$$

or (since $c_\tau \leq 0$)

$$\frac{m}{c} \leq \frac{c_{\tau_m}}{c_\tau}, \tag{42}$$

where these expressions are evaluated at $\tau_m = 0$.

   Mulligan and Sala-i-Martin (1997) consider (40) for a variety of special cases that have appeared in the literature.   For example, if utility is separable in consumption and money holdings, then $c_{\tau_m} = 0$; in this case, the right side of (42) is equal to zero, while the left side is positive.   Hence, (42) cannot hold and it is optimal to tax money.

   A second case that leads to clear results occurs if $c_{\tau_m} > 0$.   In this case, the right side of (42) is negative (since $c_\tau < 0$, an increase in the consumption tax reduces consumption).   Because the left side is nonnegative, $m/c > c_{\tau_m}/c_\tau$ and money should always be taxed.   This corresponds to a case in which money and consumption are substitutes so that an increase in the tax on money (which reduces money holdings) leads to an increase in consumption.   Finally, if money and consumption are complements, $c_{\tau_m} < 0$.   The ratio c_{τ_m}/c_τ is then positive, and whether money is taxed will depend on a comparison of *m/c* and c_{τ_m}/c_τ.

   Chari, Christiano, and Kehoe (1996) examine the optimality of the Friedman rule in an MIU model with taxes on consumption, labor supply, and money.   They show htat if preferences are homothetic in consumption and money balances and separable in leisure, the optimal tax on money is zero.   When preferences satisfy these assumptions, we can write

$$u(c, m, l) = \bar{u}[s(c, m), l],$$

where *s(c,m)* is homethetic[23].   Mulligan and Sala-i-Martin (1997) show that in this case,

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[23]   Homothetic preferences imply that *s(c,m)* is homogeneous of degree 1 and that s_i is homogeneous of degree 0.   With homothetic preferences, indifference curves are parallel to each other, with constant slope along any ray;

$$\frac{m}{c} = \frac{c_{\tau_m}}{c_\tau} ,$$

so (42) implies that the optimal tax structure yields $\tau_m = 0$.

Chari, Christiano, and Kehoe relate their results to the optimal taxation literature in public finance. Atkinson and Stiglitz (1972) show that if two goods are produced under conditions of constant returns to scale, a sufficient condition for uniform tax rates is that the utility function is homothetic. With equal tax rates, the ratio of marginal utilities equals the ratio of producer prices. To see how this applies in the present case, suppose the budget constraint for the representative household takes the form

$$(1+\tau_t^c)Q_t c_t + M_t + B_t = (1-\tau_t^h)Q_t(1-l_t) + (1+i_{t-1})B_{t-1} + M_{t-1} ,$$

where M and B are the nominal money and bond holdings, i is the nominal rate of interest, Q is the producer price of output, and τ^c and τ^h are the tax rates on consumption (c) and hours of work $(1-l)$. In addition, we have assumed that the production function exhibits constant returns to scale and that labor hours, $1-l$, are transformed into output according to $y = 1-l$. Define $P \equiv (1+\tau^c)Q$. Household real wealth is $w_t = (M_t + B_t)/P_t = m_t + b_t$, and the budget constraint can be written as

$$\begin{aligned}
c_t + w_t &= \left(\frac{1-\tau_t^h}{1+\tau_t^c}\right)(1-l_t) + (1+r_{t-1})b_{t-1} + \frac{m_{t-1}}{1+\tau_t} \\
&= (1-\tau_t)(1-l_t) + (1+r_{t-1})w_{t-1} - \left(\frac{i_{t-1}}{1+\pi_t}\right)m_{t-1}
\end{aligned} \tag{43}$$

where $1-\tau_t \equiv (1-\tau_t^h)/(1+\tau_t^c)$ and $(1+r_{t-1}) = (1+i_{t-1})/(1+\pi_t)$, and $\pi_t = P_t/P_{t-1} - 1$. Thus, the consumption and labor taxes only matter through the composite tax τ, so without loss of generality, set the consumption tax equal to zero. If the representative household's utility during period t is given by $\bar{u}[s(c,m),l]$ and the household's objective is to maximize $E_t \sum_{i=0}^{\infty} \beta^i \bar{u}[s(c_{t+i}, m_{t+i}), l_{t+i}]$ subject to the budget constraint given by (43), then the first order

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$u_2(c,m)/u_1(c,m) = f(m/c)$

conditions for the household's decision problem imply that consumption, money balances, and leisure will be chosen such that

$$\frac{\overline{u}_m(c_t,m_t,l_t)}{\overline{u}_c(c_t,m_t,l_t)} = \frac{s_m(c_t,m_t)}{s_c(c_t,m_t)} = \frac{i_t}{1+i_t} \equiv \tau_{m,t} \, .$$

With the production costs of money assumed to be zero, the ratio of marginal utilities differs from the ratio of production costs unless $\tau_{m,t} = 0$. Hence, with preferences that are homothetic in c and m, the Atkinson-Stiglitz result implies that it will be optimal to set *the nominal rate of interest equal to zero.*

Correia and Teles (1999) consider other cases in which (41) holds so that the optimal tax on money equals zero. They follow Milton Friedman (1969) in assuming a satiation level of money holdings m^* such that the marginal utility of money is positive for $m < m^*$ and nonpositive for $m \geq m^*$. This satiation level can depend on c and l. Correia and Teles show that the optimal tax on money is zero if $m^* = \overline{k}c$ for a positive constant $\overline{k}$. They also show that the optimal tax on money is zero if $m^* = \infty$. Intuitively, at an optimum, the marginal benefit of additional money holdings must balance the cost of the marginal effect on government revenues. This contrasts with the case of normal goods, where the marginal benefit must balance the costs of the marginal impact on the government's revenue *and* the marginal resource cost of producing the goods. Money, in contrast, is assumed to be costless to produce. At the satiation point, the marginal benefit of money is zero. The conditions studied by Correia and Teles (1999) ensure that the marginal revenue effect is also zero.

We can recover Friedman's rule for the optimal rate of inflation even in the absence of lump-sum taxes. But it is important to recognize that the restrictions on preferences necessary to restore Friedman's rule are very strong and, as discussed by Braun (1991), different assumptions about preferences will lead to different conclusions. The assumption that the ratio of the marginal utilities of consumption and money is independent of leisure can certainly be questioned. However, it is very common in the literature to assume separability between leisure, consumption, and money holdings. The standard log utility specification, for example, displays this property and so would imply that a zero nominal interest rate is optimal.

1.7 Nonindexed Tax Systems

Up to this point, our discussion has assumed that the tax system is indexed so that taxes are levied on real income; a one-time change in all nominal quantities and the price level would leave the real equilibrium unchanged. This assumption requires that a pure price change have no effect on the government's real tax revenues or the tax rates faced by individuals and firms in the private sector. Most actual tax systems, however, are not completely indexed to ensure that pure price-level changes leave real tax rates and real tax revenue unchanged. Inflation-induced distortions generated by the interaction of inflation and the tax system have the potential to be much larger than the revenue-related effects on which most of the seigniorage and optimal inflation literature has focused. Feldstein (1998) provides and analysis of the net benefits of reducing inflation from 2% to zero[24], and he concludes that for his preferred parameter values, the effects due to reducing distortions related to the tax system are roughly twice those associated with the change in government revenue.

One important distortion arises when nominal interest income, and not real interest income, is taxed. After-tax real rates of return will be relevant for individual agents in making savings and portfolio decisions, and if nominal income is subject to a tax rate of τ, the real after-tax return will be

$$r_a = (1-\tau)i - \pi = (1-\tau)r - \tau\pi ,$$

where $i = r + \pi$ is the nominal return and r is the before-tax real return. Thus, for a given pretax real return r, the after-tax real return is decreasing in the rate of inflation.

To see how this distortion affects the steady-state capital-labor ratio, consider the basic MIU model with an income tax of τ on total nominal income. Nominal income is assumed to include any nominal capital gain on capital holdings:

$$Y_t \equiv P_t f(k_{t-1}) + i_{t-1}B_{t-1} + P_t T_t + (P_t - P_{t-1})(1-\delta)/k_{t-1} .$$

The representative agent's budget constraint becomes

$$(1-\tau)Y_t = P_t c_t + P_t k_t - P_t(1-\delta)k_{t-1} + (B_t - B_{t-1}) + (M_t M_{t-1}) ,$$

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[24] Feldstein allows for an upward bias in the inflation rate, as measured by the consumer price index, so that his estimates apply to reducing consumer price inflation from 4% to 2%.

where M is the agent's nominal money holdings, B is his bond holdings, and $P_t T_t$ is a nominal transfer payment[25]. In real terms the budget constraint becomes[26]

$$(1-\tau)\left[f(k_{t-1}) + \frac{i_{t-1}b_{t-1}}{1+\pi_t} + T_t \right] - \tau\left(\frac{\pi_t}{1+\pi_t} \right)(1-\delta)k_{t-1}$$

$$= c_t + k_t - (1-\delta)k_{t-1} + \left(b_t - \frac{b_{t-1}}{1+\pi_t} \right) + \left(m_t - \frac{m_{t-1}}{1+\pi_t} \right).$$

Assuming the agent's objective is to maximize the present discounted value of expected utility, which depends on consumption and money holdings, the first order conditions for capital and bonds imply, in the steady state,

$$(1-\tau)f_k(k) + \left[\frac{1+(1-\tau)\pi}{1+\pi} \right](1-\delta) = \frac{1}{\beta} \tag{44}$$

and

$$(1-\tau)\left(\frac{1+i}{1+\pi} \right) + \frac{\tau}{1+\pi} = \frac{1}{\beta}. \tag{45}$$

The steady-state capita-labor ratio is determined by

$$f_k(k^{ss}) = \left(\frac{1}{1-\tau} \right)\left\{ \frac{1}{\beta} - \left[\frac{1+(1-\tau)\pi}{1+\pi} \right](1-\delta) \right\}.$$

Because $[1+(1-\tau)\pi]/(1+\pi)$ is decreasing in π, k^{ss} is decreasing in the inflation rate. Higher inflation leads to larger nominal capital gains on existing holdings of capital, and since these are taxed, inflation increases the effective tax rate on capital.

Equation (45) can be solved for the steady-state nominal rate of interest to yield

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[25] For simplicity, assume that T is adjusted in a lump-sum fashion to ensure that variations in inflation and the tax rate on income leave the government's budget balanced. Obviously, if lump-sum taxes actually were available, the optimal policy would involve setting $\tau = 0$ and following Freidman's rule for the optistate capital stock in the easiest possible manner.

[26] This formulation assumes that real economic depreciation is tax deductible. If depreciation allowances are based on historical nominal cost, a further inflation-induced distortion would be introduced.

$$1 + i^{ss} = \frac{1}{\beta}\left(\frac{1+\pi}{1-\tau}\right) - \frac{\tau}{1-\tau}.$$

Thus, the pretax real return on bonds, $(1+i)/(1+\pi)$, increases with the rate of inflation, implying that nominal rates rise more than proportionately with an increase in inflation.

It is important to recognize that we have examined only one aspect of the effects of inflation and the tax system[27]. Because of the taxation of nominal returns, higher inflation distorts the individual's decisions, but it also generates revenue for the government that, with a constant level of expenditures (in present value terms), would allow other taxes to be reduced. Thus, the distortions associated with the higher inflation are potentially offset by the reduction in the distortions caused by other tax sources. As noted earlier, however, Feldstein (1998) argues that the offset is only partial, leaving a large net annual cost of positive rates of inflation. Feldstein identifies the increased effective tax rate on capital that occurs because of the treatment of depreciation and the increased subsidy on housing associated with the deductibility of nominal mortgage interest in the United States as important distortions generated by higher inflation interacting with a nonindexed tax system. Including these effects with an analysis of the implications for government revenues and, consequently, possible adjustments in other distortionary taxes, Feldstein estimates that a 2% reduction in inflation (from 2% to zero) increases net welfare by 0.63% to 1.10% of GDP annually. These figures assume an elasticity of savings with respect to the after-tax real return of 0.4 and a deadweight loss of taxes of between 40 cents for every dollar of revenue (leading to the 0.63% figure) and $1.50 per dollar of revenue (leading to the 1.01% figure). Since these are annual gains, the present discounted value of permanently reducing inflation to zero would be quite large.

1.8 Summary

Monetary and fiscal actions are linked through the government's budget constraint. Under Ricardian regimes, changes in the money strock or its growth rate will require some other variable in the budget constraint – taxes, expenditures, or borrowing – to adjust. With fiscal dominance, changes in government taxes or expenditures can require changes in inflation. Under non-Ricardian regimes, changes in government debt affect prices even if monetary policy is exogenous. A complete analysis of price level determinacy requires a specification of the relationship between fiscal and monetary policies.

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[27] Feldstein, Green, and Sheshinski (1978) used a version of Tobin's money and growth model (Tobin 1965) to explore the implications of a nonindexed tax system when firms use both debt and equity to finance capital.

Despite this and despite the emphasis budget relationships have received in the work of Sargent and Wallace and the work on the fiscal theory of the price level initiated by Sims and Woodford, much of monetary economics ignores the implications of the budget constraint. This is valid in the presence of lump-sum taxes; any effects on the government's budget can simply be offset by an appropriate variation in lump-sum taxes. Traditional analyses that focus only on the stock of high-powered money are also valid when governments follow a Ricardian policy of fully backing interest-bearing debt with tax revenues, either now or in the future. In general, though, we should be concerned with the fiscal implications of any analysis of monetary policy, since changed in the quantity of money that alter the interest payments of the government have implications for future tax liabilities.

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