Chapter 6  Public Goods and Bads

6.1  Introduction

A public good is a commodity for which use of a unit of the good by one agent does not preclude its use by other agents. In short, a pure public good is nonrival\(^1\) in consumption. This means that once the good is provided, the additional resource cost of another person consuming the goods is zero. Knowledge provides a good illustration. The use of a piece of knowledge for one purpose does not preclude its use for others. In contrast, a private good is rival in consumption.

Several characteristics are noted.

1. Even though everyone consumes the same quantity of the good, this consumption need not be valued equally by all.
2. Classification as a public good is not an absolute (i.e. consumption of an impure public good is to some extent rival).
3. The notion of excludability\(^2\) is often linked to that of public goods.
4. A number of things that are not conventionally thought of as commodities have public good characteristics (e.g. honesty, trust and fair distribution of income).
5. Private goods are not necessarily provided exclusively by the private sector (e.g. medical services and housing).
6. Public provision of a good does not necessarily mean that it is also produced by the public sector.

For a simple exposition, an increase of one unit in the consumption of private goods reduces the consumption available to others by one unit. An increase in the consumption of public goods leads to no reduction for others. Figure 1 illustrates these relationships.

\(^1\) Nonrival is defined as consumption of the public good by one household does not reduce the quantity available for consumption by any other.

\(^2\) Non excludability is defined as follows: if the public good is supplied, no household can be excluded from consuming it except, possibly, at infinite cost.
Table 1 provides the list of goods that are commonly, but not necessarily universally, publicly provided. In each case, one can ask whether exclusion is feasible (at reasonable cost), what are the properties of demand, what are the costs of supplying to the individual, and whether these are likely to be distributional arguments.

### Table 1 Characteristics of Publicly Supplied Goods

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<th>Demand irresponsive</th>
<th>Low cost of individual supply</th>
<th>Distributional arguments</th>
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<td>National defense</td>
<td>Yes</td>
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<td>Roads and Bridges</td>
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<td>--Civil cases</td>
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<td>National parks</td>
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6.2 The Optimality Condition of Samuelson³

Consider an economy where only two goods exists: a private good $x$ and a public good $z$. The consumer $i = 1, \cdots, n$ has a utility function $U_i(x_i, z_i)$. The public good is produced from the private good according to a technology given by $z = f(x)$, where $f$ is increasing and

³ This part is drawn heavily from Salanié (2000), Chapter5.
concave. The initial resources of the economy boil down to \( X \) units of the private good.

Assume that a quantity \( x \) of private good is set aside to produce a quantity \( z \) of public good. Then, the scarcity constraint for the private good expresses that the sum of consumptions must not exceed what remains of the private good, or

\[
\sum_{i=1}^{n} x_i \leq X - x
\]  

(1)

The consumption of \( i \) in public good is limited only by the total disposable quantity since the public good is by definition \textit{nonrival}. One gets,

\[
z_i \leq z \quad \forall i = 1, \cdots, n
\]  

(2)

One obtains the Pareto optima by fixing the utility of the last \( n-1 \) consumers and by maximizing the utility of \( i=1 \) under the feasibility constraints, or

\[
\max U_i(x_i, z_i) \\
U_i(x_i, z_i) \geq \bar{U}_i \quad \forall i = 2, \cdots, n
\]

\[
\sum_{i=1}^{n} x_i \leq X - x
\]

(3)

\[
z_i \leq z \quad \forall i = 1, \cdots, n
\]

\[
z \leq f(x)
\]

Recall that in this program \( x \) represents the quantity of the private good set aside for the production of the public good and \( z \) the quantity of the public good produced. Since utility functions are increasing, one gets \( z_i = z = f(x), \forall i = 1, \cdots, n \). Let us denote \( g \) the cost in the private good for the production of the public good:

\[
z \leq f(x) \iff x = g(z)
\]  

(4)

g is an increasing and convex function. The program is simplified to,

\[
\max U_i(x_i, z_i) \\
U_i(x_i, z_i) \geq \bar{U}_i \quad \forall i = z, \cdots, n
\]

\[
\sum_{i=1}^{n} x_i \leq X - g(z)
\]

(5)
The Lagrangean can be written,

\[ L = U_i(x_i, z) + \sum_{i=2}^{n} \lambda_i (U_i(x_i, z) - \bar{U}_i) + \mu (X - g(z) - \sum_{i=1}^{n} x_i) \]  

(normalizing \( \lambda_i = 1 \) to make the formulas symmetrical, the first-order conditions are

\[ \sum_{i=1}^{n} \lambda_i \frac{\partial U_i}{\partial z} = g'(z) \]

\[ \lambda_i \frac{\partial U_i}{\partial x_i} = \mu, \quad \forall i = 1, \ldots, n \]  

(7)

From the second group of the above conditions,

\[ \lambda_i = \frac{\mu}{\partial U_i / \partial x_i} \]  

(8)

where, by substituting in the first condition the Pareto-optimality condition which sets the level of public good production,

\[ \sum_{i=1}^{n} \frac{\partial U_i}{\partial z} = g'(z) = \frac{1}{f''(x)} \]  

(9)

This is called the optimality condition of Samuelson (1954).

\[ \frac{\partial U_i}{\partial z} = \left. \frac{\partial x_i}{\partial z} \right|_{U_i} \]  

(10)

Notice that is simply the marginal rate of substitution of consumer \( i \), that is, his propensity for sacrificing his private good consumption to instigate growth is the level of his public good consumption. But as an increase of public good production by definition benefits all consumers, its marginal cost \( g'(z) \) must be compared to the sum of all propensities for paying for a public good (\( \Sigma \) (marginal rates of substitution) = the marginal rate of transformation).
6.3 Public Goods and Market Failures

In order to isolate the role of *excludability* and *rivalrousness* in consumption, we consider instances in which a good has one property but not the other. For some goods, consumption is non-rival but exclusion is possible (e.g. TV show via pay-TV). Charging a price for a non-rival good prevents some people from enjoying the good, even though their consumption of the good would have no marginal cost. Thus, charging for a non-rival good is inefficient because it results in *underconsumption*. The marginal benefit is positive, the marginal cost is zero. The *underconsumption* is a form of inefficiency.

But if there is no charge for a non-rival good, there will be no incentive for supplying the good (Is it true for free software without any compensation?). In this case, inefficiency takes the form of *undersupply*.

Thus, there are two basic forms of market failure associated with public good: *underconsumption* and *undersupply*. In the case of non-rival goods, exclusion is undesirable because it results in underconsumption. But without exclusion, there is the problem of undersupply.

1) Paying for Public Goods

If exclusion is possible, even if consumption is non-rival, governments often charge fees, called *user fees*, to those who benefit from a publicly provided good or service. Toll roads are financed by user fees. The airline ticket tax can be considered as a user fee. However, when consumption is non-rival, user fee introduces an inefficiency.

2) The Free Rider Problem

Many of the most important publicly provided goods have the property of non-excludability, making rationing by the price system unfeasible. For example, the international vaccine program against smallpox virtually wiped out the disease, to the benefit of all, whether they contributed to supporting the program or not.

The infeasibility of rationing by the price system implies that the competitive market will not generate a Pareto efficient amount of the public good. If every individual believe that he would benefit from the services provided regardless of whether he contributed to the service, he would have no incentive to pay for the services *voluntarily*.

That is why individuals must be forced to support these goods through taxation. The reluctance of individuals to contribute voluntarily to the support of public goods is referred to as...
the free rider problem (think of fire departments and lighthouses).

On the empirical ground, there are good reasons to doubt the importance of the free rider problem.

First, honesty is a social norm that molds the behavior of individuals. Second, at least in a small group, where each consumer has a notable influence on the level of public good, it is difficult for each agent to calculate the best way to undervalue its demand. Moreover, the majority of decisions of public goods production are made by elected representatives who may have less tendency to announce levels that are very low.

6.4 Optimal Provision of Public Goods and Its Finance

To simplify notation in what follows, we will take the private good as the numéraire; that is, its price will be normalized to one.

1) The Subscription Equilibrium

The first solution to consider consists of asking consumers to subscribe part of their wealth to contribute to public good production \( z \). Assume that the wealth of consumer \( i \) is \( R_i \). He can subscribe \( s_i \) to public good production, thus consuming \( x_i = R_i - s_i \) private good units.

The total quantity of public good produced will be simply \( f \left( \sum_{i=1}^{n} s_i \right) \).

The choice of \( i \) of the quantity \( s_i \) is made according to the principles of the Nash equilibrium: \( i \) will take the quantity subscribed by other consumers \( s_i \) as given and will resolve the program.

\[
\begin{align*}
\max_{x_i, s_i, z} & \quad U_i(x_i, z) \\
\text{s.t.} & \quad x_i + s_i = R_i \\
& \quad z = f \left( \sum_{i=1}^{n} s_i \right)
\end{align*}
\]

(11)

(6.11) can be simplified as

\[
\begin{align*}
\max_{s_i} U_i \left[ R_i - s_i, f \left( s_i + \sum_{j \neq i} s_j \right) \right]
\end{align*}
\]

(12)

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4 This section is also drawn heavily from Salanié (2000) Chapter 5.
This leads to the first order condition,

$$\frac{\partial U_i}{\partial z} = \frac{1}{f'(\sum_{j=1}^{h} s_j)}$$

which does not coincide with the optimality condition of Samuelson (6.10). In effect, when a consumer decides to subscribe to the public good, he takes into account only the increase of his own consumption of public good. He neglects the subsequent growth of the utility of all other consumers, so the equilibrium cannot be optimal. Under reasonable conditions, subscription equilibrium leads to a subproduction of public good.

2) Voting Equilibrium

An alternative procedure is to ask the agents to vote for their preferred level of public good. To simplify things, suppose that the public good is produced at a constant marginal cost, that is, $g'(z) = c$ for every $z$. Furthermore, suppose that each consumer believes it is his statement that will determine the level of public good production, and that the production cost will then be distributed equally among all consumers. Then each consumer chooses $z_i$ in such a way as to maximize.

$$F_i(z_i) = U_i\left(R_i - \frac{cz_i}{n}, z_i\right)$$

The voting equilibrium consists of choosing the median agent’s preferred level of production. With $m$ as the median agent, the retained level of production is $z_m$ such that $F'_m(Z_m) = 0$. So after these immediate substitutions we have

$$\frac{\partial U_m}{\partial z} = \frac{1}{f'(\sum_{j=1}^{h} s_j)}$$

This result again does not coincide with the optimal condition of Samuelson (6.10). Contrary to the subscription equilibrium, the voting equilibrium does not necessarily lead to a subproduction of the public good: the direction of the comparison depends on fine characteristics of the distribution of the marginal rates of substitution.

3) The Lindahl Equilibrium

Assume that personalized prices for the public good can be established. Every consumer $i$
must pay \( P_i \) per unit of public good that he consumes. The producer of the public good would perceive a price \( P = \sum_{i=1}^{n} P_i \) and produce up to the level where his marginal cost equals \( P \):

\[
g'(z) = P
\]  

(16)

Every consumer chooses to equate his marginal substitution rate to his personalized price:

\[
\frac{\partial U_i}{\partial z} = P_i
\]  

(17)

At equilibrium the amount of public good in demand by each consumer must equal the amount produced, or

\[
z_i(P^*) = z(P^*) \quad \forall i = 1, \ldots, n
\]  

(18)

From this, the first order condition will be

\[
\sum_{i=1}^{n} \frac{\partial U_i}{\partial z} = \sum_{i=1}^{n} P_i = P = g'(z)
\]  

(19)

This time, the optimal condition of Samuelson (6.10) is verified. The disadvantage to this process is that it assumes as a matter of fact the existence of \( n \) “micromarkets” upon which a sole consumer buys the public good at his personalized price. In such circumstances it is difficult to maintain the competitive hypothesis unless one can assume that the consumers are divided in homogeneous groups from the point of view of their propensity to pay for the public good. In the opposite case, it is in every consumer’s interest to underestimate his demand, hoping that the others will be more honest and that the level of produced public good will then be high enough to meet his needs; this leads to the free-rider problem.

4) Personalized Taxation

Suppose that every consumer \( i \) is taxed by the state in terms of his consumption \( z_i \) of public good: the budgetary constraint of the consumer \( i \) then becomes

\[
\frac{\partial U_i}{\partial z} = t_i(z_i)
\]  

(20)
If the state chooses taxes of the form \( t_i(z_i) = P_i^* z_i \), where \( P_i^* \) is the personalized price of \( i \) in the Lindahl equilibrium, it is clear that the condition of optimality will hold.

This operation assumes that the state is privy to very detailed information about the tastes of all consumers, which is not realistic. In practice, the financing of a public good is accomplished by fiscal resources from taxes and bonds. Insofar as these taxes and bonds bring on economic distortions and affect agent’s decisions, the optimality condition of Samuelson must be modified. This second-best problem reduces the optimum production level of the public good.

5) The Pivot Mechanism

Consider an indivisible public good of which 0 or 1 unit can be consumed and for which the unit costs \( C \). The utilities of the agents are assumed to be quasi-linear: if \( x_i \) represents the consumption of the sole private good and \( z \) that of the public good, we get

\[
U_i(x_i, z) = x_i + u_i z
\]

where \( u_i \) is a parameter of propensity to pay for the public good which is known only by consumer \( i \), who has initial resources \( R_i \) in private good at his disposal.

The decision to build a bridge brings a benefit of \( \sum_{i=1}^{n} u_i \) and a cost of \( C \). In the Pareto optimum the bridge should be built if and only if the sum of propensities to pay exceeds the bridge’s cost: \( \sum_{i=1}^{n} u_i \geq C \).

A first mechanism consists of asking consumers to vote on the opportunity of building the bridge, knowing that each will contribute equally to its financing.

Then the consumer \( i \) will vote for the construction if and only if \( u_i \geq \frac{C}{n} \). Let \( F \) be the cumulative distribution function of the characteristics \( u_i \) in the population. The bridge will be constructed if and only if \( F(\frac{C}{n}) \geq \frac{1}{2} \), that is if \( \frac{C}{n} \) does not exceed the median of \( F \).

The comparison with the Pareto optimal decision rule immediately shows that this mechanism is only optimal if the median of \( F \) coincides with its average, which has no particular reason to be true.

A second revealing mechanism that implements the optimal decision rule can be considered as an application of the theory of auctions. Assume an indivisible object is proposed to buyers \( i = 1, \cdots, n \) whose propensities to pay \( u_i \) are known only to themselves. First, consider a first price auction, where the winner (the one who bid the highest) pays the price he indicated.
If any consumer \( i \) announces a price \( v_i \), he will have a utility \( u_i - v_i \) if he wins and zero if not. He can guarantee himself a positive utility in expectation by announcing a price \( v_i \) inferior to his true disposition to pay \( u_i \), for he can win the bid if the other consumers are not very tempted by the object.

The second price auction suggested by Vickrey consists of making the winner (who is still the one who indicated the highest price) pay the price indicated by the person immediately after him. Consider still consumer \( i \), and let \( v_i \) be the highest price announced by the other consumers. If \( i \) announces \( v_i > v_j \), he will win the bid and will have a utility \( u_i - v_i \); if he bids \( v_i < v_j \), he will lose and will have a zero utility.

Thus he will choose to bid some \( v_i > v_j \) if \( u_i > v_i \) and some \( v_i < v_j \) if \( u_i < v_i \). In both cases his utility is independent of his bid, and it is not in his interest to lie.

Interpretation of this result goes as follows. If \( P \) is the price at which the seller values the object, the social surplus is \( u_i - P \). But if the other consumers tell the truth, the increase of utility of \( i \) in the second price auction when he raises his bid in order to win is \( u_i - u_i \), which coincides with the increase in the social surplus. The second price auction is a revealing mechanism because it leads consumers’ objectives to align themselves on the social objective.

To simplify the notation, let us subtract from the \( u_i \) the per capita cost \( c/n \), the new \( u_i \) can therefore be either positive or negative, and the Pareto-optimal decision rule amounts to constructing the bridge when \( \sum_{i=1}^{n} u_i \geq 0 \). Let \( d(u) \) be the indicator of that event.

Social surplus, with which the utility of each consumer must be identified, is \( \sum_{i=1}^{n} u_i d(u) \).

Let \( v = (v_1, \cdots, v_n) \) be the statements of the agents. In equilibrium, all consumers must tell the truth; the consumer \( i \) evaluates the social surplus at

\[
\left( \sum_{j \neq i} v_j + u_i \right) d(v)
\]

If a transfer \( t_i(v) \) is deducted from him, we then have

\[
R_i - t_i + u_i d(v) = \left( \sum_{j \neq i} v_j + u_i \right) d(v)
\]

which implies
\[ t_i(v) = -\sum_{j \neq i} v_j d(v) \]  

(24)

In fact, we can add to the transfers of every agent any quantity independent of his statement. This mechanism is characterized by

1. \( d(v) = 1 \), the bridge is constructed if and only if
\[ \sum_{j \neq i} v_j \geq 0 \]

2. The consumer \( i \) pays a transfer
\[ t_i(v) = -\sum_{j \neq i} v_j d(v) + h_i(v_{-i}) \]

where \( h_i(v_{-i}) \) is any sum depending only on statements of the other consumers.

This mechanism is revealing in dominant strategies. The utility of \( i \) under this mechanism is written,
\[ R_i + \left( \sum_{j \neq i} v_j + u_i \right) d(v) - h_i(v_{-i}) \]  

(25)

which depends on the statement \( v_j \) of \( i \) only through \( d(v) \). The agent \( i \) must choose his statement in such a way as to maximize the second term, which gives
\[ d(v) = 1 \quad \text{if and only if} \quad \sum_{j \neq i} v_j + u_i \geq 0 \]

But by definition
\[ d(v) = 1 \quad \text{if and only if} \quad \sum_{j \neq i} v_j \geq 0 \]

and \( v_i = u_i \) is one of the solutions of the program of agent \( i \), whatever the statements of the other agents may be.

Since the agent \( i \) plays a decisive role in his statement, he is called a pivot agent and this mechanism is called the pivot mechanism.

### 6.5 Public Bads or Negative Externalities

There are many public bads and negative externalities. When an agent’s actions or harm the
possibilities of other agents. For example, pollution constitutes a negative production externality, noise or cigarette smoke and negative consumption externalities.

1) The Pareto Optimum
Consider an example that comprises two goods (1 and 2), two firms and one consumer. Good 1 is polluting and good 2 is nonpolluting.
Firm 1 produces good 1 from good 2: \( y_1 = f(x_2) \).
The consumer has a utility function: \( U(x_1^c, x_2^c) \)
Firm 2 produces good 2 from good 1, since it is downstream from firm 1 (which ejects its wastes into river A) and from consumer 1 (who pollutes river B), its production suffers from two negative externalities, so its production function is, \( y_2 = g(x_1, y_1, x_1^c) \).
Denote \((w_1, w_2)\) the initial resources of the economy the Pareto optima of this economy are given by the following program:

\[
\begin{align*}
\text{Max} & \quad U(x_1^c, x_2^c) \\
\text{subject to} & \quad \begin{cases}
x_1^c + x_1 & \leq w_1 + y_1 \\
x_2^c + x_2 & \leq w_2 + y_2 \\
y_1 & \leq f(x_2) \\
y_2 & \leq g(x_1, y_1, x_1^c)
\end{cases} \quad (26)
\end{align*}
\]

where \(\lambda_1, \lambda_2, \mu_1, \mu_2\) are four multipliers.
From the first-order conditions, eliminating the multipliers,

\[
\frac{dU}{dx_1} + \left(\frac{\partial U}{\partial x_2}\right)\frac{\partial g}{\partial x_1^c} = \frac{\partial g}{\partial y_1} = \frac{1}{\frac{\partial f}{\partial x_2}} - \frac{\partial g}{\partial y_1} \quad (27)
\]

The left-hand member is the marginal rate of substitution of the consumer, corrected by the fact that his consumption of good 1 entails pollution downstream. This is called the social marginal rate of substitution of the consumer, and it takes into account all of the consequences of his consumption on social welfare. The right-hand member is the marginal rate of transformation of firm 1, corrected by the effect of its pollution on firm 2. The central term is the marginal rate of transformation of firm 2, which coincides its private marginal rate of transformation as firm 2 does not pollute.

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\(\text{Needless to say, there are many positive externalities. For example, the network effects tied to the extension of a telephone network are positive consumption externalities. Scientific research also constitutes a positive externality.}\)
The principle is that, in the presence of external effects, the usual optimality condition of equality between the marginal rates of substitution of consumers and the marginal rates of transformation of firms bears on the *social* values of these quantities. The social values take into account the external effects of each agent’s decisions on the rest of the economy.

At the optimum, firm 1 must produce less, and the consumer must consume less of good 1 than in the absence of externalities.

In 1972, the member countries of the Organization for Economic Cooperation and Development (OECD) adopted the Polluter-Pays Principle (PPP). According to this principle, “Public measure are necessary to reduce pollution and to reach a better allocation of resources by ensuring that prices of goods depending on the quality and/or quantity of environmental resources reflect more closely their relative scarcity and that economic agents concerned react accordingly...”

The Principle means that the polluter should bear the expenses of carrying out the above mentioned measures decided by public authorities to ensure that the environment is in an acceptable state.

In other words, the cost of these measures should be reflected in the cost of the goods and services which cause pollution in production and/or consumption”, (OECD, 1975).

2) Implementing the Optimum

There are several ways to internalize externalities.

*The Competitive Equilibrium*

Assume that firm 2 takes the pollution factors \( y_1 \) and \( x_1^c \) as given. Then if the prices of goods are \( p_1 \) and \( p_2 \), the agent’s choices will lead to the following relation which is inefficient.

\[
\frac{\partial U}{\partial x_1} = \frac{p_1}{\partial x_2} = \frac{1}{\partial f/\partial x_2} \quad (28)
\]

In competitive equilibrium the agents only take into account the consequences of their choices on their own welfare, in equating the private marginal rates of substitution and of transformation. Under usual conditions, one can show that there is too much of good 1 being produced and consumed.
Quotas

The simplest way to arrive at a Pareto optimum is of course for the government to set quotas specifying that the externality-inducing activities should be set at their optimal level. This is certainly an authoritarian solution and one that assumes a very fine knowledge of the characteristics of the economy. In the example studied here, it would amount to calculating the Pareto-optimal levels of $y_1$ and $x_1^c$, which we denote $y_1^*$ and $x_1^{c*}$, and to forbid firm 1 from producing more than $y_1^*$ and the consumer from consuming more than $x_1^{c*}$ of good 1. It is nevertheless an oft-adopted solution, under a slightly less brutal form, that consists of limiting the quantity of a certain type of pollutants emitted by firms or even by consumers (as in carbon emissions of automobiles).

Subsidies for Depollution

It is sometimes possible to install dispositions to reduce pollution. Suppose that firm 1 can invest in depolluting a quantity $a$ of good 2 which, without affecting its production, reduces its pollution as if $y_1$ were $y_1 - d(a)$.

First, let us look for the Pareto optimum. The scarcity constraint for good 2 becomes

$$x_2^c + x_2 + a \leq \omega_2 + y_2$$  \hspace{1cm} (29)

while the production constraint of firm 2 becomes

$$y_2 \leq g[x_1, y_1 - d(a), x_i^c]$$  \hspace{1cm} (30)

The quantity $a$ also becomes a maxim and of the program. We easily see that the optimality conditions obtained above are unchanged, but that we must add to them a condition to determine the optimal depollution level $a^*$:

$$d'(a^*) = -\frac{1}{\partial g/\partial y_i}$$  \hspace{1cm} (31)

What happens here with the competitive equilibrium? If firm 1 is not prompted to depollute, it will of course choose not to do so. It still seems reasonable for the government to subsidize such an expense by paying firm 1 a sum $s(a)$. The profit maximization program of firm 1 then becomes

$$\max_{x_2,a}(p_1 f(x_2) - p_2 x_2 - p_2 a + s(a))$$  \hspace{1cm} (32)
while firm 2 maximizes in $x_i$

$$p_2 g[x_i, y_i - d(a), x_i^c] - p_i x_i$$  \hfill (33)

The condition of profit maximization of firm 1 implies that

$$s'(a) = 1$$  \hfill (34)

So choosing subsidy $s(.)$ such that $s'(a^*) = 1$ induces the firm to realize the socially optimal expenditures of depollution. Unfortunately, the first-order conditions also entail

$$\frac{\partial g}{\partial x_i} = \frac{1}{\frac{\partial f}{\partial x_2}}$$  \hfill (35)

So the subsidized equilibrium is still not Pareto optimal.

The Rights to Pollute

Meade (1952) suggested a solution to the problem of external effects which has generally found favor with economists (and more rarely with decision makers). It rests on the finding that externalities contribute to inefficiency only because no other market exists upon which they may be exchanged. Therefore assume that the state (or any other institution) creates a “rights to pollute” market: a right to pollute gives the right to a certain number of pollution units. The polluters (firm 1 and the consumer) can then pollute with the proviso that they buy the corresponding rights to pollute. Thus

- firm 1 pays $q$ to firm 2 for each unit of $y_1$
- the consumer pays $r$ to firm 2 for each unit of $x_i^c$

The consumer’s program gives

$$\frac{\partial U}{\partial x_1} = \frac{p_1 + r}{p_2}$$  \hfill (36)

while the profit maximization of firm 1 implies that
\[
\frac{p_1 + q}{p_2} = \frac{1}{\partial f / \partial x_2} \quad (37)
\]

As for firm 2, it must take into account payments it receives for determining the number of pollution rights it is ready to sell; it solves therefore

\[
\max_{x_1, y_1, x_i^c} \left[ p_2 g(x_1, y_1, x_i^c) - p_1 x_1 + r x_i^c + q y_1 \right] \quad (38)
\]

whence

\[
\begin{cases}
\frac{p_1}{p_2} = \frac{\partial g}{\partial x_1} \\
\frac{q}{p_2} = -\frac{\partial g}{\partial y_1} \\
\frac{r}{p_2} = -\frac{\partial g}{\partial x_i^c}
\end{cases} \quad (39)
\]

All of these equalities amount to

\[
\left( \frac{\partial U}{\partial x_1} \right) + \left( \frac{\partial U}{\partial x_2} \right) \left( \frac{\partial g}{\partial x_1} \right) = \frac{\partial g}{\partial x_1} = \frac{1}{\partial f / \partial x_2} - \frac{\partial g}{\partial y_1} \quad (40)
\]

which is the condition of efficiency. The creation of markets for rights to pollute thus implements a Pareto optimum\(^7\). In other respects, it is a system far less demanding than the imposition of pollution quotas, since the calculation of such quotas implies that the state knows the preferences and technologies of all agents. It is enough here for the state to open pollution rights markets and let equilibrium establish itself\(^8\).

This result merits some comments. First, note that the consumer and firm 1 do not necessarily pay the same price to pollute at equilibrium. We would get \(q = r\) only if the pollution was impersonal in the sense that \(g(x_1, y_1, x_i^c) = G(x_1 + x_i^c, y_1)\). Second, there is

\(^7\) Neglected here are second-order conditions, which Starrett (1972) has shown are problematic. In effect, function \(g\) cannot be concave in \(y_1\) or \(x_i^c\) on all of \(\mathbb{R}^+\), since it is decreasing and positive. The program of firm 2 is therefore nonconvex, and this can cause difficulties for equilibrium.

\(^8\) We assume that the pollutees are authorized to sell pollution rights, which results in an optimal pollution level. In practice, these markets are often reserved to the polluters. The pollution level attained depends then on the number of rights put into circulation, which poses the problem of the government’s capacity to calculate the optimal pollution level, to issue the correct number of rights to pollute, and to resist the pressure of agents who would like to see that number modified.
one sole applicant and one sole supplier on each open pollution rights market in this example, which raises the problem of strategic behaviors. This solution is therefore better adapted to situations where the pollution is of collective origin – we can imagine similarly disposed polluters around the same lake\(^9\).

**Taxation**

It is conceivable to tax the production of the good \(1\) at the rate \(\tau\) and its consumption at the rate \(t\). It is easy to see that if one chooses \(\tau = q\) and \(t = r\), where \(q\) and \(r\) are the equilibrium prices on virtual markets of pollution rights, we again find a Pareto optimum. These tax levels are often called *Pigovian taxes*, in honor of Pigou (1928). The disadvantage to this remedy is that like the imposition on the pollution quotas, one needs extraordinarily detailed information on the primitive data of the economy, since the government must be able to calculate the equilibrium prices on pollution rights markets.

**The Integration of Firms**

For simplicity’s sake assume that the consumer does not pollute, so \(\frac{\partial g}{\partial x^t_i} = 0\). In this case one can envision the two firms as merging (in economic terms, we speak of “intergrating”). The new firm thus formed will maximize the joint profit as

\[
\max_{x_1, x_2, y_1} \left\{ p_1 f(x_2) + p_2 g(x_1, y_1) - p_2 x_2 - p_1 x_1 \right\}
\]

\(y \leq f(x_2)\)  \hspace{1cm} (41)

which gives

\[
\frac{p_1}{p_2} = \frac{\partial g}{\partial x_1} = \frac{1}{\partial f/\partial x_2} - \frac{\partial g}{\partial y_1}
\]

Again we have the Pareto optimum. The solution is obviously radical, and it shows little regard for property rights. We often see in industrial economics that the market power conferred upon mastodons is not without inconveniences. Nevertheless, the integration of firms is not to be discarded entirely.

\(^9\) One of the most spectacular applications of the pollution rights market functions in the San Francisco bay area (see Henry 1989). The regulation of thermal power stations and of sulfur dioxide emissions in the United States offers other examples. More recently the summit on global warming held in Kyoto in 1997 decided to study the use of rights markets at the world level.
**Must Prices or Quantities Be Regulated?**

We have seen that regulation by quantities (e.g., in the form of emission quotas) and regulation by prices (e.g., by taxation) both permit the restoration of Pareto-optimality of the equilibrium and are therefore equivalent. Let us follow Weitzman (1974) and consider a pollutant production $q$. The firm has costs $C(q)$ and its profit is then $\left( pq - C(q) \right)$ if the price is $p$. The production entails "benefits" $B(q)$ and a consumer surplus $B(q) - pq$.

This modeling suggests two comments. The first is of a semantic nature: many times students are amazed that pollutant production can be beneficial. In reality there is often joint production of a useful good and of pollution; what we call "benefits" of the pollutant production is the value of useful production minus the cost inflicted by pollution. The difference between benefits and costs is generally maximal for a positive level of pollution.

The reader will note that in this interpretation\(^\text{10}\), the social production cost at once comprises the production cost $C(q)$ and the pollution cost, which was deducted from the value of production to get $B(q)$.

The second comment is more technical. In the example we have studied so far $q$ would be $y_1$, the production of firm 1. This production permits the consumer to raise his consumption of good 1, but it reduces the production possibilities of firm 2. The sum of these two effects constitutes the "benefits" of $y_1$, which has moreover a cost given by the technology of firm 1. Unfortunately, it is not possible to describe the result in the form of a benefit minus a cost. Such a breakdown is in fact more reasonable when the pollution injures consumers. In the interest of not straying too far from our needs, we will disregard this consideration here. We will also make the usual hypotheses that $B(q)$ is increasing and concave and $C(q)$ increasing and convex.

When information is perfect, the optimal emission quota is calculated by maximizing the social surplus $\left( B(q) - C(q) \right)$. We then get

$$B'(q^*) = C'(q^*) \quad (43)$$

As for the optimal tax rate, it is fixed in such a way that the prices verify

$$p^* = B'(q^*) = C'(q^*) \quad (44)$$

Then the firm effectively produces the optimal pollution level $q^*$.

At this stage the two modes of regulation are perfectly equivalent. Still, opinions on their respective advantages are generally quite clear-cut. As noted by Weitzman (1974, p.477):

\(^{10}\) We could resort to a dual interpretation where $q$ represents the nonpolluted good, like air quality.
I think it is a fair generation to say that the average economist in the Western marginal tradition has at least a vague preference toward indirect control by prices, just as the typical noneconomist leans toward the direct regulation of quantities.

The introduction of an imperfection of government information on the costs and benefits will let us compare the two modes of regulation. Thus suppose that either because the agents benefit from private information or because the future is uncertain, the cost and benefit functions are affected by independent shocks \( \theta \) and \( \eta \) so that they become \( C(q, \theta) \) and \( B(q, \eta) \).

Ideally price or quantity would be fixed conditionally to realizations of shocks in such a way as to verify

\[
p^*(\theta, \eta) = \frac{\partial B}{\partial q}[q^*(\theta, \eta), \eta] = \frac{\partial C}{\partial q}[q^*(\theta, \eta), \theta] \tag{45}
\]

and regulations by prices and by quantities would remain strictly equivalent.

In practice, this first-best solution is beyond reach, since the government must make decisions knowing only the distributions of \( \eta \) and \( \theta \) and not their realizations. In this second-best situation, the government chooses the emissions quota \( \hat{q} \) so as to maximize the expected social surplus

\[
E(B(q, \eta) - C(q, \theta)) \tag{46}
\]

Things are only slightly more complicated for regulation by price. If the government fixes a price \( p \), the firm will choose a production \( Q(p, \theta) \) such that

\[
p = \frac{\partial C}{\partial q}[Q(p, \theta), \theta] \tag{47}
\]

The government must then fix the price at the level \( \hat{p} \) that maximizes

\[
E[B[Q(p, \theta), \eta] - C[Q(p, \theta), \theta]] \tag{48}
\]

This time there is no more reason that the two modes of regulation be equivalent. In fact it is easily checked (using second-order calculations for small uncertainties) that the advantage (in
terms of expected social surplus) of regulation by prices on regulation by quantities is

\[
\Delta \approx \frac{\sigma^2}{2(c^\alpha)^2} \left( \frac{\partial^2 B}{\partial q^2} + \frac{\partial^2 C}{\partial q^2} \right) \tag{49}
\]

where \( \sigma^2 \) is the variance of the marginal cost \( \partial C/\partial q \). Since \( B \) is concave and \( C \) convex, the sign of \( \Delta \) is ambiguous a priori. Note that if marginal costs are almost constant, then regulation by quantities will dominate regulation by prices. In effect a small error in price setting can lead to a large error in the level of pollution attained.

### 6.6 Coase’s Theorem

In a famous article Coase (1960) doubted the necessity of any governmental intervention in the presence of externalities. His reasoning is very simple: let \( b(q) \) be the benefit that the polluter draws from a level of pollutant production \( q \) and \( c(q) \) the cost thus imposed on the pollutee\(^{11}\). When \( b \) is concave and \( c \) increasing and convex, the optimal pollution level is given by

\[
b'(q^*) = c'(q^*) \tag{50}
\]

Suppose that the status quo \( q_0 \) corresponds to a situation where \( b'(q_0) < c'(q_0) \), and thus the pollution level is too high. Then the polluter and the pollutee have an interest in negotiating. Let \( \varepsilon \) be a small positive number, and assume that the polluter proposes to lower the pollution level to \( (q_0 - \varepsilon) \) against a payment of \( t\varepsilon \), where \( t \) is comprised between \( b'(q_0) \) and \( c'(q_0) \). Since \( t > c'(q_0) \), this offer raises the polluter’s profits; and it is equally beneficial for the pollutee, since \( t < b'(q_0) \). Therefore the two parties will agree to move to a slightly lower pollution level. The reasoning does not stop here: so long as \( b' < c' \), it is possible to lower the pollution level against a well-chosen transfer from pollutee to polluter. The end result is the optimal pollution level. A very similar argument applies in the case where \( b'(q_0) > c'(q_0) \). The “Coase theorem” can thus be set forth as follows:

If property rights are clearly defined and transaction costs are zero the parties affected by an externality succeed in eliminating any inefficiency through the simple recourse of negotiation.

\(^{11}\) Be careful not to identify this notation with that of the preceding section.
Stated in this way, the theorem becomes more a tautology: if nothing keeps the parties from negotiating in an optimal manner, they will arrive at a Pareto optimum. In fact Coase (1988) explained in a collection of his articles that above all he wanted at the time to bring attention to the importance of property rights and of transaction costs. Unfortunately, it is more than anything his “theorem” that has passed into posterity.

What happens if the hypotheses of the theorem do not hold? First of all note that in many industrial pollution cases, property rights are not defined. For a common resource like ocean fish, for example, it is impossible to identify the polluters and the pollutees and then put a negotiation into place; Coase’s theorem is therefore of no great help to us in attacking overfishing. Even if property rights are clearly defined, transaction costs are rarely negligible. For example, these costs include expenses incurred during negotiation (lost time, necessary recourse to lawyers, etc.). In the argument above, an elementary negotiation improves the social surplus by \( (c'(q_0) - b'(q_0)) \epsilon \). If the salary of the retained lawyer is higher, the parties will renounce this stage of the negotiation and will stop before having attained the optimal level.

Recent literature has above all insisted on the transaction costs due to asymmetrical information\(^{12}\). If the polluter has private information on \( b'(q_0) \) and the pollutee has private information on \( c'(q_0) \), each will try to “hog the blanket”, thereby manipulating the transfer \( t \). Myerson-Satterthwaite (1983) show that under these conditions the parties will not be able to achieve a Pareto optimum. When the concerned parties become more numerous, it is of course much more difficult for each to them to manipulate this “prices” \( t \). In fact the negotiation becomes again asymptotically efficient when the number of agents tends toward infinity (Gresik-Satterthwaite 1989). One can still wonder about the capacity of a large number of polluters and pollutees to negotiate together, since other transaction costs then risk coming out.

These critiques do not reduce the interest in the Coase theorem to nothing. In fact Cheung (1973) shows that in the case of the beekeeper-orchard crossexternality made popular by Meade, there exist in the United States contracts between two parties that seek to internalize the externality. It is therefore reasonable to think that under certain conditions the private agents, left to themselves, can effectively negotiate to arrive at a level of externality that, if not optimal, is at least more satisfying. The imperfections of such a solution to the externality problem must in any case be compared to those of the Pigovian solutions in a world where governmental information is quite imperfect.

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\(^{12}\) Farrell (1987) offers a good discussion of this topic.
Exercises

1. Under the United Nations framework convention on climate change, the global warming problem has been discussed and the UN tries to reach some agreements on the limit to greenhouse gas around the world.

   Could you identify the difficulties to solve the global warming problem, given the different stages of economic development? Could you provide some solutions for this?
References


