Chapter 5  Capital Income Taxation

5.1 Taxation of Income from Capital

The Ramsey rule for efficient taxation provides a justification for taxing income from capital differently from income from labor, since governments are advised to set tax rates inversely proportional to supply elasticities. That is, with \( t_L \) the rate of taxation of labor income and \( t_K \) the rate of taxation of income from capital, the Ramsey rule proposes that tax rates be set so that

\[
\frac{t_L}{t_K} = \frac{\varepsilon_{SK}}{\varepsilon_{SL}}
\]

where \( \varepsilon_{Si} (i = L, K) \) is the supply elasticity. The supply elasticities in expression (1) reflect opportunities to leave a tax jurisdiction. If capital can readily leave, the elasticity of supply of capital \( \varepsilon_{SK} \) is high. Conversely, if labor cannot leave, the elasticity of supply of labor \( \varepsilon_{SL} \) is low. The Ramsey rule therefore implies that the rate of taxation on income from capital should be lower than the rate of taxation on income from labor. If capital in a tax jurisdiction is part of a broader capital market and can simply leave in response to a tax on income from capital, the tax rate on capital indicated by the Ramsey rule is zero.

The home bias in investment

Possibilities for taxing capital depend on the willingness of investors to hold assets outside their tax jurisdiction. Investors often seem reluctant to hold foreign assets, which results in a “home bias” in asset holdings. The home bias may be due to investors’ believing that they are better informed about investments in their home markets. Investors may also trust their own

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2 This statement of the Ramsey rule is based on infinite elasticities on the other side of the market (here demand elasticities) and independence between markets so that a tax in one market does not affect tax revenue in other markets.
3 Personal income taxes usually have more than one rate. In expression (1) for the Ramsey rule, there is one rate of taxation for income depending on the source of income. That is, we are not now asking questions about the structure of taxation for different levels of income. Nor are we asking questions about different rates of taxation for different people implied by the Ramsey rule.
4 If the broader market, for example, offers a return of 5 percent, any attempt to tax capital to reduce the local return below 5 percent will lead investors to move their capital outside the jurisdiction where they can obtain the 5 percent return.
government more than foreign governments to protect their ownership rights. The home bias reduces the supply elasticity of capital and, through the Ramsey rule, increases the efficient tax on income from capital relative to income from labor.

**Social justice**

A government may believe that social justice requires taxing income from capital at a higher rate than income from labor. The presence of capital markets that extend beyond the government’s tax jurisdiction, however, limits the scope for taxing income from capital. If capital can leave the jurisdiction to escape the tax, a government has no choice but to set low taxes on income from capital and high taxes on income from labor because, by attempting to tax income from capital, the government ends up with little or no capital to tax.

**Portfolio investment and real assets**

When people choose to save rather than consume, resources can be invested to create capital. The capital can be human capital invested in the skills, ability, and knowledge of a person or can take the form of physical capital. When we refer to movement of capital out of a tax jurisdiction, we may mean physical capital, and also human capital. Machinery and equipment move as firms relocate their operations to other tax jurisdictions, and high human-capital people might also move to escape taxes.

People are also described as investing when they buy stocks or shares in a company, or when they buy government or corporate bonds. Such portfolio investment changes the ownership of assets but does not create new assets. Where people hold their personal investment portfolios affects their tax obligations, through the different taxes that different government levy in their jurisdictions.

Where people “hold” their asset portfolios is a legal fiction. For example, a portfolio of shares in U.S., European, and Japanese companies can be “held” in a Caribbean island where low taxes (or no taxes) are levied on income from portfolio investments of nonresidents. The government of the Caribbean island gains tax revenue by attracting offices of “off-share” portfolio management companies and through legal fees.

Locations with low or no taxes on nonresident! Income from portfolio investment are non as tax havens\(^5\). The presence of tax havens limits other governments’ abilities to tax income from ownership of capital. When taxes on income from capital increase, no actual capital may leave the tax jurisdiction, but an asset “portfolio” may leave, by the transfer of the location of ownership of the asset portfolio to a tax haven.

We therefore need to distinguish between investment as financial capital and investment as

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\(^5\) The financial press lists information about the values of shares in investment companies in tax havens.
physical capital. In both cases, capital can be mobile. However, portfolio investment is more mobile than physical investment.

**Residence-based taxation**

To avoid loss of tax revenue by transfer of assets to foreign locations, governments may define tax liability based on the location of residence of the taxpayer and not on the location of the office of the investment company that holds assets in the name of the taxpayer. However, even with residence-based taxation, assets can be held abroad beyond the reach of the taxation authorities, if foreign banks maintain a policy of confidentiality regarding identities of account holders. Taxpayers are then engaging in tax evasion by holding asset portfolios in foreign tax havens.

**Dynamic inconsistency**

Taxation of income from capital is subject to the problem of dynamic inconsistency. To explain this problem, we observe that, even though financial capital is in general very mobile, the physical capital in place after an investment has been made is less mobile. Often, after an investment in physical capital has been made (a factory has been built), the capital cannot readily move elsewhere. The supply elasticity of capital after the investment has been made is then zero. The supply elasticity of capital before the investment is made, may however be, quite high because of alternative locations where the investment can be made and also because of the option not to make the investment at all.

Therefore, the supply elasticity of capital before the investment decision has been made is in general greater than after the investment has been made. A government that wishes to apply the Ramsey rule for efficient taxation could announce a low rate of taxation of income from capital before an investment is made, to reflect the high supply elasticity of capital. However, after the investment has been made, the Ramsey rule calls for a high rate of taxation on income from the investment because of the subsequent low (or zero) supply elasticity of capital.

Announcement by a government of a low rate of taxation on income from capital is therefore *dynamically inconsistent*. The low tax announced today will not be the tax rate that the government will wish to apply tomorrow. The announcement of a low tax rate before the investment has been made will therefore not be credible to investors because investors know that it is in the interest of the government to increase the tax rate after an investment has been made. If the announcement of a low tax rate is not credible, investments will not be made, because of the anticipation by investors of high taxes in the future.

There is a saying that the best tax is an old tax. That is, it is wise for government not to change taxes. Stable taxes allow investment decisions to be made without the uncertainty of
having to predict future government tax policy. Investment is by nature subject to uncertainty. Uncertainty about taxes increases the uncertainty that firms face when deciding whether to invest. A reputation for stable taxes is a way of establishing and confirming a commitment not to take advantage of capital that becomes immobile after an investment has been made.

Therefore, there are benefits from maintaining an unchanged rate of taxation on income from capital. When tax rates on income from capital are increased, the credibility that tax rates will not be changed again in the future is lost, and investors can become reluctant to invest.

Similar considerations affect taxation of natural resources. After an exploration company has discovered an oil deposit, the income earned from the oil is a rent because there is no way to use the asset other than to extract oil. The Ramsey rule calls for appropriative taxes on rents because rents are returns from investments for which supply elasticities are zero. We see the time-inconsistency problem about taxation intentions that confronts an oil exploration company. The elasticity of supply of capital for oil exploration is high, which by the Ramsey rule calls for low tax rates on future income from successful exploration. However, after oil is found, the Ramsey rule calls for high taxes. Because of the time-inconsistency problem, the rule that an old (stable) tax is a good tax also applies to incentives for oil exploration, and exploration for other natural resources.

**The Effects of Taxation on Savings**

In most countries, income taxation bears both on labor income and on income from savings. With perfect financial markets, taxation of labor income only changes the savings rate in that the latter depends on permanent income. We will focus here on the effect of taxation of income from savings on the time profile of consumption over the life cycle. We will therefore start by neglecting the taxation of labor income. We also assume an exogenous interest rate, which neglects general equilibrium effects.

**Theoretical Analysis**

Consider a consumer who lives two periods and whose labor supply is inelastic. He gets in the first period a wage \( w \), consumes some part of it, and saves the rest according to

\[
C_t + E = w
\]

In the second period, he does not work and consumes the net income from his savings. Given an interest rate \( r \) and taxation of income from savings at a proportional rate \( t \), his budget

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6 This part draws from Salanié (2003) Chap 2, pp.45-9.
7 Thus the first period could be the active period of life and the second retirement.
constraint in the second period is

\[ C_2 = (1 + r(1 - t))E \]

Assuming perfect financial markets, the consumer can save as much as he likes, and the two budget constraints can be aggregated in an intertemporal constraint

\[ C_1 + pC_2 = w \]

where \( p \) is the relative price of second-period consumption, that is,

\[ p = \frac{1}{1 + r} \]

without taxation and

\[ p = \frac{1}{1 + r(1 - t)} \]

with taxation.

As usual, the increase in \( p \) due to taxation has two effects:

- an income effect: the increase in \( p \) reduces both \( C_1 \) and \( C_2 \) if consumptions in both periods are normal goods, which increases savings \( E = w - C_1 \)
- a substitution effect: the increase in \( p \) makes second-period consumption more expensive and thus tends to reduce savings.

More precisely, denote \( U(C_1, C_2) \) the utility function of the consumer. We can write

\[
\frac{\partial C_1}{\partial p} = \left( \frac{\partial C_1}{\partial p} \right)_U - C_2 \frac{\partial C_1}{\partial w}
\]

Define the intertemporal elasticity of substitution as

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8 We can neglect the disutility of labor since labor supply is assumed to be inelastic.
\[ \sigma = \left( \frac{\partial \log(C_1/C_2)}{\partial \log p} \right)_U \]

First note that since Hicksian demands are the derivatives of the expenditure function \( e(p,U) \) with respect to prices, the equality

\[ C_1(p,U) + pC_2(p,U) = e(p,U) \]

implies by differentiating in \( p \) that

\[ \left( \frac{\partial C_1}{\partial p} \right)_U + p \left( \frac{\partial C_2}{\partial p} \right)_U = 0 \]

Since by definition

\[ \sigma = \left( \frac{\partial \log C_1}{\partial \log p} \right)_U - \left( \frac{\partial \log C_2}{\partial \log p} \right)_U \]

we obtain

\[ \sigma = \left( \frac{p}{C_1} + \frac{1}{C_2} \left( \frac{\partial \log C_1}{\partial \log p} \right)_U \right) = \frac{w}{pC_2} \left( \frac{\partial \log C_1}{\partial \log p} \right)_U \]

and

\[ \left( \frac{\partial \log C_1}{\partial \log p} \right)_U = e\sigma \]

where \( e = E/w = pC_2/w \) denotes the savings rate.

We also have
\[ \frac{\partial C_1}{\partial w} = C_1 \frac{\partial \log C_1}{w \log w} \]

Finally, by substituting within the Slutsky equation and denoting

\[ \eta = \frac{\partial \log C_1}{\partial \log w} \]

the income elasticity of first-period consumption, we get

\[ \frac{\partial \log C_1}{\partial \log p} = e\sigma - C_1 \frac{p}{C_1} \frac{\partial \log C_1}{w} = e(\sigma - \eta) \]

Moreover

\[ \frac{\partial \log E}{\partial \log p} = -\frac{C_1}{E} \frac{\partial \log C_1}{\partial \log p} \]

whence

\[ \frac{\partial \log E}{\partial \log p} = -(1 - e)(\sigma - \eta) \]

which shows the negative substitution effect \((-1-e)\sigma\) and the income effect \((1-e)\eta\).

What is the order of magnitude of the resulting effect? Note that once more, the Cobb-Douglas utility function is not much help since it implies \(\sigma = \eta = 1\) and thus no effect of taxation on savings. A reasonable assumption is that preferences are homothetic so that both consumptions are proportional to permanent income \((\eta = 1)\). Choose \(e = r = 1/2\), which is not absurd since the two periods represent the working life and retirement. Then a 50 percent tax on income from savings increases \(p\) by 20 percent and reduces savings by 10 percent multiplied by \((\sigma - 1)\) (to the first order). Thus, to get large effects of taxation on savings, the intertemporal elasticity of substitution has to be rather large. It is even quite possible that taxation increases savings (it is the case if and only if \(\sigma < \eta\)).

If the consumer is paid wages in both periods, then we must take into account a new income effect as permanent income becomes.
This time the consumer may decide to borrow (if his second-period wages are relatively high), which makes imperfections on financial markets relevant. If the interest rate at which he can borrow $r^+$ is larger than the interest rate paid on his savings $r^-$, then his budget constraint has a kink at the zero savings point. Under these circumstances some consumers will choose to locate in that point\(^9\), and the substitution effect does not come into play, at least locally. This clearly reduces the negative influence of taxation on savings.

So far we have neglected the taxation of labor income. If it is taxed at the same rate as income from savings (as is the case for the ideal income tax), $w$ must be replaced with $w(1-t)$. Then taxation reduces permanent income and thus both consumptions. Since savings this time is $w(1-t) - C_1$, the way this effect goes depends on the income elasticity $\eta$.

The taxation of savings affects not only income but also accumulated savings. Such is the case for wealth taxes, but also for taxes on bequests. Assume that in addition to his consumptions the consumer derives utility from any (after-tax) bequest $H$ he leaves at his death. Then his utility is $U(C_1, C_2, H)$, and given a taxation rate $\tau$ on bequests, his second-period budget constraint becomes

$$C_2 + \frac{H}{1-\tau} = E(1 + r(1-t))$$

His intertemporal budget constraint becomes

$$C_1 + pC_2 + p'H = w$$

where $p$ is still defined as

$$p = \frac{1}{1+r(1-t)}$$

and $p' = p/(1-\tau)$. With a taxation rate of income from savings fixed at $p$, by the Hicks-Leontief theorem, the two consumptions can be aggregated within a composite good. The effect of changes on the rate of bequest taxation $\tau$ then is formally analogous to that of $t$.

\(^9\) They are liquidity constrained: they consume their income within each period.
on savings. This analysis of bequests is only half convincing, however. Whether bequests are planned or accidental (due to early deaths) is a controversial issue. In any case the taxation of bequests, like wealth taxes, collects very small amounts of tax revenues in most countries.

**Empirical Results**

In the 1970s econometricians tried to estimate the elasticity of aggregate consumer savings to the after-tax interest rate. Apart from Boskin (1978) who obtained a value close to 0.4, most estimates were close to zero. This quasi-consensus was shaken by a paper of Summers (1981). Using the calibration of a life-cycle model in a growing economy, Summers showed that any choice of parameters compatible with the observed ratio of wealth to income implied a large elasticity of savings to the interest rate. More recent work, however, has showed that Summers’s result is fragile.

The literature turned in the 1980s to the estimation of Euler equations derived from the intertemporal optimization of consumers; this yielded values for the intertemporal elasticity of substitution $\sigma$. Studies done on macroeconomic data have yielded small values for $\sigma$. More credible estimations on individual data suggest that $\sigma$ is nonnegligible but lower than one (which is its value for a Cobb-Douglas utility function), which implies a very small elasticity of savings to the interest rate.

Finally, many authors have used the existence of investments that are favored by taxation. Most of these studies use data from the United States, where it is possible to use Individual Retirement Accounts (IRA) and 401(k) funds to save into pension funds and deduct the amount saved from taxable income. These funds have been very successful, but the important question is whether the money that went into them would have been saved anyway or not. The studies are not unanimously conclusive, but it seems that total savings was only moderately stimulated by the favorable tax treatment of these funds.

A general lesson of this literature is that taxation is unlikely to have a large impact on total savings, although it clearly plays an important role in determining where the money is invested.

**Taxation and Risk-Taking**

Taxation is often said to discourage risk-taking, since it confiscates part of the return to risky activities such as setting up a business or investing in shares. Domar and Musgrave (1944) however noted that taxation transforms government into a sleeping partner who absorbs part of the risk, which may in fact encourage risk-taking. We will revisit this argument following

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10 Bernheim (2002) contains a much more detailed discussion.
11 This part draws from Salanié (2003) Chap 2, pp.49-52.
Mossin. We will set aside the question of whether such risk-taking as exists in the economy is too large or too small – popular opinion is that risk-taking is insufficient and should be encouraged, but there is no good evidence either for or against this view.

We consider the portfolio choice of investing in a riskless asset that brings a return \( r \) and a risky asset that brings a random return \( x \). We therefore assume that there exists a safe asset in the economy, which is an approximation (even the purchasing power of money is affected by inflation). We also assume that two-fund separation holds: all risky assets may be aggregated in a single composite risky asset\(^\text{13}\).

The investor has a strictly concave von Neumann-Morgenstern utility function \( u \) on wealth strictly concave, meaning that he is risk-averse. We denote \( W_0 \) the initial wealth invested in the risky asset, then

\[
W = (1 - \tau)W_0 (1 + (ax + (1 - a)r)(1 - t))
\]

where \( \tau \) is the tax rate on wealth and \( t \) is the tax rate on asset income.

The investor maximizes \( Eu(W) \) in \( a \), which gives the first-order condition

\[
E(u'(W)(x - r)) = 0
\]

Taxation enters this expression via final wealth \( W \). The impact of taxation of wealth is easy to see, since it just multiplies initial wealth \( W_0 \) by \( (1 - \tau) \). We know from Arrow (1970) that if absolute risk-aversion \(-u''(W)/u'(W)\) is non-increasing in wealth\(^\text{14}\), then the amount invested in the risky asset increases with wealth, which means in our case that \( a(1 - \tau) \) is a decreasing function of \( \tau \). To go further and to conclude that the proportion \( a \) invested in the risky asset is reduced by the taxation of wealth, we need to ensure that \( a \) increases with wealth. This is only true if relative risk-aversion decreases with wealth, which is not clear from the empirical evidence.

Now assume that wealth is not taxed \( \tau = 0 \) and focus on taxation of asset income. The first-order condition is

\[
E(u'(W_0(1 + (ax + (1 - a)r)(1 - t)))(x - r)) = 0
\]

\(^{12}\) To make the problem nontrivial, we assume that \( Ex > r \) and that \( r \) lies in the interior of the support of \( x \).

\(^{13}\) Two-fund separation was used for the first time by Tobin (1958). It can be justified under rather strict assumptions on preferences (see Cass and Stiglitz (1970)).

\(^{14}\) This so-called NIARA (non-increasing absolute risk-aversion) hypothesis is confirmed by almost all empirical studies.
Let us differentiate it with respect to \( t \). We get

\[
E\left[u''(W)(x - r)\left( (x - r)(1 - t) \frac{\partial a}{\partial t} - (ax + (1 - a)r) \right) \right] = 0
\]

whence by rearranging

\[
- \frac{\partial \log a}{\partial \log(1 - t)} = 1 + \frac{r}{a} \frac{Eu''(W)(x - r)}{E(u''(W)(x - r)^2)}
\]

(3)

First note an interesting special case: if \( r = 0 \) (e.g., if the riskless asset is money in a world without inflation), we find that

\[
- \frac{\partial \log a}{\partial \log(1 - t)} = 1
\]

which shows that \( a(1 - t) \) is independent of \( t \) and therefore implies that taxation increases \( a \). The intuition is that of Domar and Musgrave (1944): taxation amounts to a participation of government in risk and therefore encourages risk-taking.

When \( r \neq 0 \), things are slightly more complicated. Taxation of income from the riskless asset indeed reduces wealth and may change the attitude of the investor toward risk. To evaluate the second term in (3), we must define the amount invested in the risky asset \( Z = aW_0 \) and study how it changes with wealth. Rewriting the first-order condition (2) with this new notation, we get

\[
E(u'(W_0)(1 + r(1 - t) + (1 - t)(x - r)Z)(x - r)) = 0
\]

or by differentiating with respect to initial wealth \( W_0 \),

\[
E \left( u''(W)(x - r) \left( 1 + r(1 - t) + (1 - t) + (x - r) \frac{\partial Z}{\partial W_0} \right) \right) = 0
\]

Rearranging obtains
Substituting (3) in (2) finally yields

$$\frac{-\partial \log Z}{\partial \log W_0} = \frac{Eu''(W)(x-r)}{E(u''(W)(x-r)^2)} \frac{1+r(1-t)}{a(1-t)}$$

Therefore there is a new wealth effect that induces an increase in risk-aversion and thus makes risk-taking less appealing than in the \( r = 0 \) case. Arrow thought that the elasticity of \( Z \) in \( W_0 \) must be lower than one. If such is the case, then the right-hand side is still positive and taxation must always encourage risk-taking, but Arrow’s hypothesis is controverted.

Assume that \( r > 0 \). We saw that under the NIARA hypothesis, \( Z \) increases in \( W_0 \). Therefore there is a new wealth effect that induces an increase in risk-aversion and thus makes risk-taking less appealing than in the \( r = 0 \) case. Arrow thought that the elasticity of \( Z \) in \( W_0 \) must be lower than one. If such is the case, then the right-hand side is still positive and taxation must always encourage risk-taking, but Arrow’s hypothesis is controverted.

Note that until now we implicitly assumed that the government shared in losses as well as gains. This assumption can be justified if losses can be deducted from gains on other risky assets and the resulting gain is always positive. Otherwise, it is useful to examine the impact of the no loss offset rule, whereby the government does not subsidize losses. Then \( x(1-t) \) must be replaced with \( x \) when \( x < 0 \); with \( r \geq 0 \), it does not change the return of the riskless asset. It is easy to see that when \( t \) gets close to one, then taxation must always reduce risk-taking: it does not reduce losses and gains become negligible. In general, taking the no loss offset rule into account tends to reduce risk-taking relative to the case where the government also takes its share of the losses.

Finally, note that in most countries, capital gains are taxed at a lower rate than interest income. Then one should consider that \( x \) and \( r \) are in fact \( x(1-t') \) and \( r(1-t) \), with \( t' < t \). If risk-aversion does not vary too much with wealth, then this tends to increase risk-taking relative to a uniform taxation.

These remarks show that the real world is more complex than appears from the model. Moreover taxation of asset income is one of the most intricate areas of existing tax systems. Since household data usually are not very detailed on portfolio holdings, this makes estimating the effect of taxation on the holdings of risky assets very difficult. The survey of Poterba (2002) nevertheless concludes that taxation has substantial effects on how households allocate their wealth.
The Taxation of Capital\textsuperscript{15}

Labor income only comprises about two-thirds of GDP, while one-third goes to capital. Taxation of capital therefore deserves its own analysis. It is a very vast subject, since many things go under the name of capital. The common element to all of these is that capital is accumulated savings, whether it be physical capital (machines and buildings used for production, but also housing held by families) or financial capital (bank and financial assets, e.g., bonds and shares). Thus taxation of capital in fact involves two sorts of taxes:

- taxes on the stock of capital like the wealth tax, the tax on bequests, property taxes
- taxes on the income from savings, such as the corporate income tax\textsuperscript{16}, taxation of interest and dividends, and the taxation of capital gains\textsuperscript{17}.

The economic analysis of these two cases in fact is very similar, since capital stocks stem from accumulated savings. Thus the distinction will play no role in this chapter.

As in the last three chapters, the standpoint adopted here is that of optimal taxation. Thus our main concern here is with the optimal level of capital taxation. However, we will also return to the incidence of capital taxation. In the whole chapter we neglect considerations lined to risk. This makes the analysis much less realistic but also much more tractable.

In many European countries the press and some politicians complain that the taxation rate of capital goes down while taxation of labor income seems to increase inexorably. This is deemed unfair, since a common (but simplistic) theory of incidence suggests that workers pay the tax on labor income and capitalists pay the tax on capital. This is also thought to encourage substitution of capital for labor, thus reducing labor demand. Therefore the relative decline of the taxation of capital would be partly responsible for high unemployment and low wages.

As often in the theory of taxation, theoretical results are quite remote from popular discourse. We will see in this chapter that at least to a first approximation, there is no strong reason to tax capital at all: the optimal tax rate on capital is zero! This a priori surprising result actually is an old theme in economic literature. It is linked to the optimality of a consumption tax. As is easily seen by writing the intertemporal budget constraint of a worker-consumer who receives and leaves no bequest,

\[ \sum_{t=1}^{T} \frac{C_t}{(1 + r)^t} = \sum_{t=1}^{T} \frac{w_t L}{(1 + r)^t} \]

\textsuperscript{15} This part draws heavily from Salanié (2003) Chap6, pp.121-40.
\textsuperscript{16} Although the question of what the corporate income tax actually taxes is controversial.
\textsuperscript{17} The stock-flow distinction is not very clear when it comes to capital gains; however, they are usually taxes as income.
a tax on consumption is equivalent to a tax that bears on labor income only, to the exclusion of income from savings.

**Applying Classical Results**

The effect of taxation of income from savings is to increase the price of future consumption relative to that of current consumption. Thus what matters is whether we should tax future consumption more than current consumption. The results seem to apply directly to this problem, since we may consider consumptions at various dates as many as different goods.

Ramsey’s formula gives us a first guide. With a representative consumer, the Corlett-Hague result shows that a higher tax on future consumptions may be in order if they are more complementary to leisure than current consumption. This in principle is an empirically testable proposition: Does a permanent wage increase translate (for fixed utility) into an increase of future consumptions smaller than that in current consumption? Unfortunately, we have no convincing answer to this question. If future consumptions and current consumption are equally complementary to leisure but individuals are heterogeneous, then Ramsey’s formula indicates that future consumption should be discouraged more if the rich tend to defer their consumption more than the poor. Observation shows convincingly that the rich have a higher propensity to save than the poor, which seems to provide us with a second argument for taxing capital.

However, Ramsey’s model in fact is very restrictive, as it only allows for proportional labor income taxation. Now turn to the Atkinson-Stiglitz result on the role of indirect taxation with an optimal nonlinear income tax. This theorem tells us that if the utility function of consumers is weakly separable, so that it can be written

\[
U(C_1, \ldots, C_T, L_1, \ldots, L_T, w) = \bar{U}(h(C_1, \ldots, C_T), L_1, \ldots, L_T, w)
\]

then all consumptions should be taxed at the same rate and income from savings should not be taxed at the optimum. Under this separability hypothesis, the targeting principle applies: it may well be that the rich save more, but this is because they have higher incomes, and that comes from their higher productivity. Thus the optimal trade-off between equity and efficiency can be attained by taxing only labor income.

The fact that the Atkinson and Stiglitz result neglects inherited wealth seems the most relevant here: if consumer-workers come to life with different productivities and different bequests, then the latter should be taxed at the optimum. There is indeed no relevant difference between endowments at birth, whether they consist in higher productivity or a higher bequest. However, this argument
neglects the effect of capital taxation on the decisions of the person who leaves the bequest. Taxation of bequests becomes more problematic if they are planned by altruistic donors.

**Capital Accumulation**

For simplicity, we assume that every generation has only one type of agent: redistributive considerations are not essential here since we focus on the capital accumulation process. As before, generations grow at rate \( n \); labor supply is now assumed to be inelastic (each young person supplies one unit of labor). Capital does not depreciate, and there is no technical progress. We start the analysis by assuming that there is no taxation to begin with. Recall that we are in a closed economy, so savings must directly feed investment.

With an inelastic labor supply, the representative agent of generation \( t \) maximizes \( U(C_{yt}, C_{0, t+1}) \) under his budget constraint

\[
\begin{align*}
C_{yt} + S_t &= w_t \\
C_{0, t+1} &= (1 + r_{t+1})S_t
\end{align*}
\]

where \( S_t \) is the savings of generation \( t \). We obtain the intertemporal budget constraint as

\[
C_{yt} + \frac{C_{0, t+1}}{1 + r_{t+1}} = w_t
\]

The first-order condition is

\[
\frac{U'_y(C_t)}{U'_0(C_t)} = 1 + r_{t+1}
\]

which generates demand functions \( C(w_t, r_{t+1}) \) and savings

\[
S(w_t, r_{t+1}) = w_t - C_y(w_t, r_{t+1})
\]

We still denote \( F(K, L) \) production (net of capital), and we denote \( f(k) = F(k,1) \) net production per capita. As usual, profit maximization gives factor incomes:
\begin{align*}
\begin{cases}
w_t - f(k_t) - k_t f'(k_t) \\
r_t = f'(k_t)
\end{cases}
\end{align*}

where we used the notation $k_t = K_t / L_t$.

Finally, equilibrium on the market for capital implies the equality between the savings of the young $S_t$ and the capital stock in the next period\(^{18}\). Taking into account demographic growth, this gives

$$k_{t+1}(1 + n) = S(w_t, r_{t+1}) = S(f(k_t) - k_t f'(k_t), f'(k_{t+1}))$$

This equation defines the dynamics of the capital accumulation process. Unfortunately, it is very nonlinear, which raises several difficulties. First note that $k_{t+1}$ appears on both sides of the equation, so the dynamics is only defined implicitly. We will assume here that we can in fact extract a dynamic mapping $k_{t+1} = g(k_t)$. Even then, this system may not be stable. Now assume that there exists a unique solution $k^*$ of

$$k^*(1 + n) = S(f(k^*) - k^* f'(k^*), f'(k^*))$$

This solution is locally stable if

$$\left|g'(k^*)\right| < 1$$

that is, if

$$\frac{-k^* f''(k^*) S_n}{1 + n - f''(k^*) S_r} < 1$$

Then the per capita capital stock converges (at least locally) to $k^*$, which defines the *steady state* equilibrium where capital intensity is constant and the stock of capital grows at the same rate $n$ as the population.

The graphs may illustrate this solution concept visually.

\(^{18}\) The capital stock of period $t$ is recycled, and some of it may be consumed – remember that there is only one good in the economy.
As is clear from Figure 1, a lower equilibrium point $A$ is stable while an upper equilibrium point $B$ is unstable. Figure 2 illustrates a possibility of multiple stable equilibriums, $A'$ and $C'$.

We will always assume that the steady state is unique and locally stable, so as to look at its comparative statics. Simple computations show that if the savings function also depends on a parameter $\theta$,

$$S_i = S(w_i, r_{i+1}, \theta)$$

then if the steady state is locally stable, the steady state capital intensity $k^*$ is a function of $\theta$ whose sign is that of $S'_{\theta}$: a variation of $\theta$ whose direct effect is to increase savings indeed leads
to an increase in capital intensity.

The uniqueness and local stability assumptions imposed on the dynamic system are very strong: in general, one cannot exclude indeterminacies (several solutions in $k_{t+1}$ for a given $k_t$) and/or cyclical or even chaotic dynamics.

Is the steady state optimal? Optimality is rather easy to define if we focus on stationary trajectories, where all per capita allocations are constant. Then all generations have the same utility, and we naturally seek to maximize the utility of any of them. Thus consider a stationary allocation with capital intensity $k$. Our old friend the benevolent and omniscient planner only faces one constraint: since he needs $nk$, given population growth, to keep the capital intensity unchanged, he only has $(f(k) - nk)$ to share between the old and the young in any given period. Since the old at each time are $(1 + n)$ times fewer than the young, we must therefore have

$$C_y + \frac{C_0}{1 + n} = f(k) - nk$$

If we maximize $U(C_y, C_0)$ under this constraint, obviously we first need to maximize $(f(k) - nk)$, which gives

$$r = f'(k) = n$$

At the social optimum, capital intensity must be such that the interest rate equals the rate of growth of the population. This is called the golden rule, and it defines a capital intensity $k^G$.

Can we compare the optimum $k^G$ and the steady state $k^*$? Unfortunately, there is no reason why these two capital intensities should coincide. Assume, for instance, that the utility has the Cobb-Douglas form

$$U(C_y, C_0) = a \log C_y + (1 - a) \log C_0$$

and that the production function also is a Cobb-Douglas:

$$F(K, L) = K^a L^{1-a}$$

Then
\[ f(k) = k^\alpha \]

The consumption and savings functions are very simple:

\[
\begin{align*}
C_y &= aw \\
C_o &= (1 + r)(1 - a)w \\
S &= (1 - a)w
\end{align*}
\]

The dynamics of capital intensities is

\[ k_{t+1}(1 + n) = (1 - a)(1 - \alpha)k_t^\alpha \]

which indeed converges to a steady state given by

\[ k^* = \left( \frac{(1 - a)(1 - \alpha)}{1 + n} \right)^{\frac{1}{1 - \alpha}} \]

The golden rule gives

\[ \alpha(k^G)^{\alpha-1} = n \]

or

\[ k^G = \left( \frac{\alpha}{n} \right)^{\frac{1}{1 - \alpha}} \]

It is easily seen that \( k^G \) may be smaller or greater\(^{19}\) than \( k^* \). Moreover no reasonable condition will make them equal: in general, the steady state is not optimal\(^{20}\).

When \( k^G < k^* \), one can even improve the utility of all generations. Just assume that just after

\(^{19}\) The reader may check that seems more likely with “reasonable” values of \( a, \) and \( n, \) but once again, this model is not particularly realistic.

\(^{20}\) This is no violation of the first welfare theorem: here we have an infinity of agents indexed with their date of birth, whereas the proof of the first theorem heavily relies on the assumption that the number of agents is finite.
production at some date $T$, the government decides to release $(k^* - k^G)$ for consumption and to then maintain the capital intensity at $k^G$. The quantity of good available for consumption in period $T$ is

$$f(k^*) + (k^* - k^G) - nk^G = f(k^*) - nk^* + (n+1)(k^* - k^G)$$

which is larger than $(f(k^*) - nk^*)$: both generations present at $T$ can consume more. At later dates the quantity available for consumption is

$$f(k^G) - nk^G > f(k^*) - nk^*$$

by the definition of the golden rule, and here again all generations alive at these dates can consume more. With such excess accumulation of capital, we say that the economy is dynamically inefficient: there exists a reallocation that improves the utility of all generations\(^{21}\). Otherwise, we say that the economy is dynamically efficient – which does not mean that the level of capital is optimal, since $k^* \leq k^G$.

Let us now return to the taxation of capital. Any tax (or subsidy) on capital affects savings and therefore capital accumulation. If the steady state is not optimal, such a tax can thus bring the economy closer to the golden rule. If, for instance, we tax capital at rate $\tau$ and we redistribute the tax revenue to the young (with a transfer $T_y$) and to the old (with a transfer $T_o$), then the consumer’s budget constraints become

$$\begin{cases} 
C_y + S = w + T_y \\
C_o = (1 + r(1 - \tau))S + T_o 
\end{cases}$$

or in the intertemporal form

$$C_y + \frac{C_0}{1 + r(1 - \tau)} = w + T_y + \frac{T_o}{1 + r(1 - \tau)}$$

so that savings become

\(^{21}\) Abel et al. (1989) study empirically the accumulation process in the main developed countries. They conclude that the economy is dynamically efficient in each of these countries, which suggests that they have too little capital.
\[
S = w + T_y - C_y \left( w + T_y + \frac{T_0}{1 + r(1 - \tau)} , r(1 - \tau) \right)
\]

Transfers must, of course, balance the government’s budget constraint, which is

\[
T_y + \frac{T_0}{1 + n} = \frac{r \tau S}{1 + n}
\]

since the tax collects \( r \tau S \) on each old individual and the old are \((1 + n)\) times fewer than the young in each period.

Creating such a tax changes the capital accumulation process through

\[(1 + n)k = S\]

We could pursue with complicated computations, but let us focus on the effects involved instead. The creation of the tax has, as usual, two opposite effects on savings: the income effect increases savings, and the substitution effect reduces it\(^{22}\). A transfer to the old tends to reduce savings since it helps finance second-period consumption. Finally, a transfer to the young increases savings in the reasonable case where the marginal propensity to consume is below one. There are thus five effects to account for, and two independent instruments (since \( \tau , T_y , T_0 \) are linked by the government’s budget constraint).

For simplicity, return to the case where the utility function is Cobb-Douglas; then the income effect and the substitution effect cancel out and we have

\[
S = (1 - a)(w + T_y) - \frac{aT_0}{1 + r(1 - \tau)}
\]

First assume that the revenue from taxing capital is entirely given to the young: \( T_0 = 0 \) and \( T_y = \frac{r \tau S}{(1 + n)} \). Then we get

\[
S = \frac{(1 - a)w}{1 - (1 - a)r \tau / (1 + n)}
\]

\(^{22}\) There is here a third effect due to the discounting of the transfer to the old; this tends to reduce savings if \( T_0 \) is positive.
which increases in $\tau$. If we start from a dynamically inefficient economy, then the government can move closer to the golden rule by discouraging savings. This is done here by subsidizing savings and by financing the subsidy through a lump-sum tax on the young. If the economy is dynamically efficient, the government should tax capital and transfer the revenue to the young.

The government can also move the economy closer to the golden rule without transferring money to or from the young, that is $T_y = 0$ and $T_0 = r\tau S$. Then we compute

$$\left(1 + \frac{ar\tau}{1 + r(1 - \tau)}\right)S = (1 - a)w$$

and $S$ now decreases in $\tau$. If the economy is dynamically inefficient, the government now should tax capital and transfer the revenue to the old; if it is dynamically efficient, the right policy is to subsidize savings and raise a lump-sum tax on the old.

This example, of course, is purely illustrative: Cobb-Douglas functions may not be very realistic, and the two-period model where only the young save is a caricature\(^{23}\). However, it shows that the optimal way to tax capital depends heavily on how the tax revenue is used; for any given economy, one may want to tax or subsidize savings depending on whether the government balances its budget on the old or on the young. The intuition is simple: in this model the young save and the old dissave. Thus we encourage savings by making a transfer to the old and we discourage savings by making a transfer to the young.

Diamond’s model therefore yields a rather ambiguous justification of capital taxation: the optimal policy may be a subsidy as well as a tax. In fact we will now see that the argument for taxing capital is even weaker than that: there are other ways to get closer to the golden rule, such as intergenerational transfers and the public debt.

We already implicitly studied the role of intergenerational transfers. Let us return to our analysis of capital taxation and assume that $\tau = 0$, so that capital is neither taxed nor subsidized. Then we must have

$$T_y + \frac{T_0}{1 + n} = 0$$

\(^{23}\) Several authors (e.g., see Auerbach and Kotlikoff (1987)) use for simulation purposes more complex models that may have up to eighty generations living at each date. The central mechanisms, however, are the same as in our two-generation model.
But under reasonable assumptions, we know that savings $S$ increases in $T_y$ and decreases in $T_0$ (this is clearly true for a Cobb-Douglas utility function, as we saw above). If the economy is dynamically inefficient, the government moves the economy closer to the golden rule by making a transfer from the young to the old: this discourages savings and thus reduces capital intensity. If, on the other hand, the economy is dynamically efficient, the government should make a transfer from the old to the young instead.

Note that transfers from the young to the old exist in many countries, where they finance (at least part of) Social Security and are known as pay-as-you-go pensions systems. The defining feature of such a system is that at each date, the social contributions paid by the young finance the pension benefits of the old. Thus a reasoned choice of the level of pension benefits may bring the economy closer to the golden rule.

Intergenerational transfers are not the only way to move the economy closer to the golden rule. All governments finance part of their budget deficits by issuing public debt, that is, financial assets that are bought by private agents. We distinguish two forms of public debt, which do not have the same effects: external public debt, which is held by nonresidents, and internal public debt, which is held by agents who live in the country. Let us focus on internal public debt, since the agents live in a closed economy. Assume that debt is contracted for one period, and that it pays the same interest rate as capital – which must be true in equilibrium since there is no risk. Part of the savings of the young now is invested in government bonds to finance their consumption when they are old. Let $b$ denote the per capita stock of public debt at the steady state. Since the per capita stock of debt must be constant, the government must issue exactly $nb$ in new debt at each date, and it must pay $rb$ interest on current debt. The difference $(r - n)b$ is financed by lump-sum taxes (possibly negative). If, for instance, these taxes are paid by the young, capital market equilibrium becomes

$$(k + b)(1 + n) = S(w - (r - n)b, r)$$

since savings now both funds investment and public debt. It is easy to see that public debt is a perfect substitute for intergenerational transfers. We can indeed rewrite the equation as

$$(k + b)(1 + n) = w - (r - n)b - C_y(w - (r - n)b, r)$$

which we can identify as equivalent to the capital market equilibrium with intergenerational transfers.

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Once again, this is a caricature: real-world social contributions are not lump sum. Since they are based on wages, they tend to discourage labor supply, which is inelastic in our version of Diamond’s model.
transfers:  

\[ k(1 + n) = w + T_y - C_y \left( w + T_j + \frac{T_j}{1 + r}, r \right) \]

where \( T_j \) and \( T_0 \) are defined by

\[
\begin{cases}
    T_y = -b(1 + r) \\
    T_0 = (1 + r)(1 + n)b
\end{cases}
\]

These transfers are clearly balanced at each date since \( T_y + T_0 / (1 + n) = 0 \). Thus internal public debt is equivalent to a transfer from the young to the old. The economy moves closer to the golden rule if public debt increases (resp. decreases) and the economy is dynamically inefficient (resp. efficient).

The conclusion suggested by this brief study of capital accumulation and government policy is rather mixed. As was shown by Atkinson and Sandmo (1980), whether capital taxation is of any use depends a lot on the policy instruments of government. Capital taxation may be a way to move the economy closer to the golden rule, but one may need a subsidy as well as a tax. Moreover the government may reach the same goal without taxing or subsidizing capital but simply by setting the level of pension benefits or that of internal public debt. Of course, a more realistic study of the problem should take into account the distortions on labor supply induced by the income tax or social contributions.

**The Incidence of Capital Taxation**

We already studied the incidence of capital taxation in Harberger’s model. However, that model assumes that the supply of capital is inelastic. We will now briefly look at the incidence of capital taxation when one takes into account its effects on capital accumulation.

The very stylized overlapping generations model we used until now is not adequate for this study, since it only has one consumer per generation. We adopt here the extension by Kaldor and Pasinetti of Solow’s neoclassical growth model to an economy with both workers and capitalists.

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25 This is in addition to the fact that the young already balance the government’s budget constraint.

26 This part draws from Salanié (2003, Chap6, pp.141-4).
We neglect capital depreciation and technical progress. The balanced growth equilibrium is given by

\[ s(k)f(k) = nk \]

where \( s(k) \) is the savings rate, the ratio of savings to production. Kaldor and Pasinetti endogeneized the way it depends on capital intensity by assuming the coexistence of two classes, workers and capitalists. Capitalists receive capital income \( r_k = f'(k)k \) and have a propensity to save \( s_r \), while workers are paid wages \( w = f(k) - kf''(k) \) and have a lower propensity to save \( s_w < s_r \). This savings function obviously are rather ad hoc; they allow us nevertheless to model in a very simple way the heterogeneity of private agents.

Under these assumptions

\[ s(k)f(k) = s_r f'(k)k + s_w (f(k) - kf''(k)) \]

If we introduce a tax on capital at rate \( \tau \) and redistribute the tax revenue \( \tau f'(k)k \) to workers\(^{27}\), then the equilibrium is given by

\[ s_r (1 - \tau)f'(k)k + s_w (f(k) - kf''(k) + \tau k f''(k)) = nk \]

With \( s_r > s_w \) and for fixed \( k \), the left-hand side decreases in \( \tau \). If the equilibrium is locally stable decreasing savings imply a lower equilibrium capital intensity. As a consequence the gross return of capital \( r = f'(k) \) increases and wages \( w = f(k) - kf''(k) \) decrease: the tax is partly shifted to workers (whose income still increases since they receive the proceeds of the tax). As a matter of fact, if \( s_w = 0 \), we have

\[ s_r (1 - \tau)f'(k) = n \]

and the net return of capital \( (1 - \tau)f'(k) \) is unchanged by the tax: the tax is entirely shifted to workers\(^{28}\).

This reasoning is only valid in a closed economy. The polar opposite of a closed economy is a small open economy, which cannot influence world prices. Then with perfect capital mobility, the

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\(^{27}\) It would be rather strange to tax capital and return the proceeds to capitalists.

\(^{28}\) This sentence is not completely true: production per capita and capital intensity also decrease, which reduces the income of capitalists \( r(1 - \tau)k \). On the other hand, workers have unchanged incomes, as the proceeds of the tax exactly compensate the decrease in wage income.
net return of capital cannot deviate from its world value$^{29}$:

$$f'(k)(1 - \tau) = r^*$$

Then the tax on capital is entirely shifted on workers$^{30}$: the before-tax return increases so as to leave unchanged the net return, and wages go down. Gordon (1986) relies on this as an argument against taxing capital in a small open economy: since taxing capital makes it fly abroad (which reduces capital intensity at home) and eventually is shifted to labor, it is better to tax labor directly. However, the assumption of perfect capital mobility is overdone: despite globalization, capital markets still are far from perfectly integrated.

**Conclusion**

As we have seen, there are several possible justifications for taxing capital. The first reason is that weak separability may be a bad assumption when it comes to the choice between current and future consumptions. Unfortunately, we lack empirical evidence to substantiate this claim. The second reason is that if agents inherit wealth that is not well taxes by the tax on bequests, then it may be optimal to tax capital income. Note in this regard that if these bequests were planned by the donors, then the Chamley and Judd result goes the other way.

The second justification for capital taxation resides in the sub-optimality of capital accumulation in the economy. In the likely case in which our economies suffer from too little capital, we saw that if, for instance, agents have a Cobb and Douglas utility function, it may be useful to tax capital and to transfer the tax revenue to the young. Note, however, that the same result may be achieved by raising a lump-sum tax on the old and using it to subsidize capital. Moreover one may dispense with taxing or subsidizing capital altogether and just use intergenerational transfers or change the level of the public debt.

Thus none of these arguments seem to provide a very secure footing for taxing capital (and certainly none of them supports the lay-man’s opinion that capital and labor should be taxed at the same rate). However, there is another argument. We assumed until now that the relative wages of the various individuals were exogenous. In Stiglitz’s model of capital taxation we thus assumed that the skill premium was equal to the relative productivity $v$. If this relative wage in fact depends on capital intensity, then $v = v(K_t)$. It is easy to see that the derivative of the Lagrangian in $K_{t+1}$ then would be

$^{29}$ If, for instance, the net return were lower than abroad, capital would flee the economy; this would reduce $k$ and thus increase the net return until it becomes equal to its value abroad.

$^{30}$ But see note 23.
\[
\gamma_{t+1} (1 + F' K (K_{t+1}, E_{t+1})) - \gamma_t - \lambda_{t+1} \frac{\partial U^2}{\partial L_{t+1}} v' (K_{t+1}) = 0
\]

whence

\[
\frac{\gamma_{t+1}}{\gamma_t} = 1 + F' K (K_{t+1}, E_{t+1}) - \lambda_{t+1} \frac{\partial U^2}{\partial L_{t+1}} v' (K_{t+1})
\]

Given existing empirical evidence (e.g., see Krusell et al. 2000), there are good reasons to think that unskilled labor is more substitutable to capital than skilled labor. In other words, it is easier to replace unskilled labor with machines than to do the same with skilled labor. Then an increase in the capital intensity in the economy tends to increase the relative wage of skilled labor, so that \( v(K) \) is a decreasing function. It follows immediately from the equation 31 above that the after-tax interest rate is smaller than the before-tax interest rate, meaning that capital should be taxed.

This argument is quite general: one could check that in the Chamley and Jud model, such a configuration of elasticities of substitution would also lead to taxing capital. Of course, this effect remains to be quantified. Note also that some authors think that the increasing role of computers in production could make skilled labor a better substitute for capital.

This chapter set aside some points that are nevertheless important. Thus we always assumed full employment. It is not clear how unemployment would affect our results; in any case, that certainly would depend on the causes of unemployment. We also assumed that financial markets were perfect. If some agents in fact find it easier to borrow than others, then such a market imperfection makes the allocation of capital among competing projects inefficient. In particular, if the rich find it easier to borrow, it is reasonable to think that this would justify some taxation of capital. Even without such considerations, Chamley (2001) shows that when agents are constrained on their borrowing, it may be optimal to tax capital. To see this, assume that future wages are affected by idiosyncratic shocks (independent across agents). Since it is difficult to borrow, each consumer will build up precautionary savings to guard against times when his wage will be low. In general, consumption at one given date will be positively correlated with accumulated savings. Then taxing this accumulated savings to finance a uniform lump-sum transfer effectively redistributes from high consumption states of the world to low consumption states of the world. This smooths consumption across states and thus increases the expected utilities of consumers. Once again, the tax here acts as an insurance device.

Finally, note that all that we said so far also applies to the taxation of human capital: under some conditions that we needn’t repeat, the return to human capital should not be taxed. Only the innate

31 Remember that utility decreases in \( L \).
productivity of agents should be taxed. This recommendation may seem abstract, but one can derive practical consequences from it. For a start, assume that the tax on labor income is proportional. Then a student who must decide whether to study one more year compares the value of his forgone wages to the increase in future wages that a higher diploma will give him. With a proportional tax, the tax rate appears on both sides a drop out so that the tax is neutral. In practice, the tax is usually progressive and thus it discourages the accumulation of human capital. Moreover investments in human capital are not limited to forgone wages. We should also include tuition fees, which therefore should be deductible from taxes. Similarly taxes paid to finance public education expenditures should be deductible from taxable income.
References


